Exponents and Logarithms

January 2014

Question 3 (calculator) 2 marks

An earthquake in Vancouver had a magnitude of 6.3 on the Richter scale. An earthquake in Japan had magnitude of 8.9 on the Richter Scale.

How many times more intense was the Japan earthquake than the Vancouver earthquake?

Solution

Method 1

Vancouver: substitute $M = 6.3$

$$6.3 = \log\left(\frac{A}{A_0}\right)$$

$$10^{6.3} = \frac{A}{A_0}$$ ½ mark for exponential form

$$A = 10^{6.3} A_0$$

Japan: substitute $M = 8.9$

$$8.9 = \log\left(\frac{A}{A_0}\right)$$

$$10^{8.9} = \frac{A}{A_0}$$ ½ mark for exponential form

$$A = 10^{8.9} A_0$$

To compare the two earthquakes divide their intensities.

$$\frac{\text{the intensity of Japan}}{\text{the intensity of Vancouver}} = \frac{10^{8.9} A_0}{10^{6.3} A_0}$$ 1 mark for comparison

$$= 398.107$$

$$= 398$$

2 marks

Question 32 4 marks

Solve the following equation:

$$2\log_4 x - \log_4(x+3) = 1$$

Solution

$2\log_4 x - \log_4(x+3) = 1$

$\log_4\left(\frac{x^2}{x+3}\right) = 1$ — 1 mark for power rule; 1 mark for quotient rule

$4^1 = \left(\frac{x^2}{x+3}\right)$ — 1 mark for exponential form

$4(x+3) = x^2$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6$ ~~$x = -2$~~ — ½ mark for solving for x; ½ mark for rejecting extraneous root

4 marks

Question 39 1 mark

Which of the following equations could be solved without the use of logarithms? Without actually solving the problem, explain your choice.

$$4^x = 10^{3x+1} \quad \text{or} \quad \left(\frac{1}{3}\right)^{2x+1} = 27^{4x-1}$$

Solution

$\left(\frac{1}{3}\right)^{2x+1} = 27^{4x-7}$ can be solved without the use of logarithms because $\frac{1}{3}$ and 27 can both be changed to a base of 3.

1 mark for explanation

1 mark

Question 5 (calculator) 3 marks

Given $\log_a 9 = 1.129$ and $\log_a 4 = 0.712$, find the value of $\log_a 12$.

Solution

Method 1

$$\log_a 9 = 1.129$$
$$\log_a 3^2 = 1.129$$
$$2\log_a 3 = 1.129$$
$$\log_a 3 = 0.5645$$

1 mark for power rule

$$\log_a 12 = \log_a (4 \cdot 3)$$
$$= \log_a 4 + \log_a 3$$
$$= 0.712 + 0.5645$$
$$= 1.2765$$
$$= 1.277$$

1 mark for writing 12 as a product

1 mark for product rule

3 marks

Method 2

$$\log_a 12 = \log_a \left(\sqrt{9} \cdot 4\right)$$
$$= \frac{1}{2}\log_a 9 + \log_a 4$$
$$= \frac{1}{2}(1.129) + 0.712$$
$$= 1.2765$$
$$= 1.277$$

1 mark for writing 12 as a product

1 mark for power rule
1 mark for product rule

3 marks

June 2013

Question 8 3 marks

Solve the following equation algebraically:

$$\log_3(x-4)+\log_3(x-2)=1$$

Solution

Method 1

$\log_3(x-4)+\log_3(x-2)=1$

$\log_3(x-4)(x-2)=1$ — 1 mark for product rule

$3^1=(x-4)(x-2)$ — 1 mark for exponential form

$3=x^2-6x+8$

$0=x^2-6x+5$

$0=(x-5)(x-1)$

$x=5$ ~~$x=1$~~ — ½ mark for solving for x within a quadratic equation; ½ mark for rejecting extraneous root

3 marks

Method 2

$\log_3(x-4)+\log_3(x-2)=1$

$\log_3(x-4)(x-2)=1$ — 1 mark for product rule

$\log_3(x^2-6x+8)=\log_3 3$ — ½ mark for logarithmic form

$x^2-6x+8=3$ — ½ mark for equating arguments

$x^2-6x+5=0$

$(x-1)(x-5)=0$

~~$x=1$~~ $x=5$ — ½ mark for solving for x within a quadratic equation; ½ mark for rejecting extraneous roots

3 marks

Question 30 1 mark

Which expression has a larger value?

$\log_2 36$ or $\log_3 80$

Justify your answer.

Solution

Method 1

$\log_2 36$ $\quad 2^? = 36$ $\begin{cases} 2^5 = 32 \\ 2^6 = 64 \end{cases}$ ≈ 5.1

$\log_3 80$ $\quad 3^? = 80$ $\begin{cases} 3^3 = 27 \\ 3^4 = 81 \end{cases}$ ≈ 3.9

$\therefore \log_2 36$ is the larger value

1 mark for justification

1 mark

Method 2

$\log_2 32 = 5 \therefore \log_2 36$ is a little more than 5

$\log_3 81 = 4 \therefore \log_3 80$ is a little less than 4

$\therefore \log_2 36$ is the larger value

1 mark for justification

1 mark

Question 13 3 marks

Determine the value of y in the following equation:

$$\log_x 27 - \log_x 3 = 2\log_x y$$

Solution

$\log_x 27 - \log_x 3 = 2\log_x y$

$\log_x \frac{27}{3} = 2\log_x y$ — 1 mark for quotient rule

$\log_x 9 = \log_x y^2$ — 1 mark for power rule

$9 = y^2$

$y = \pm 3$

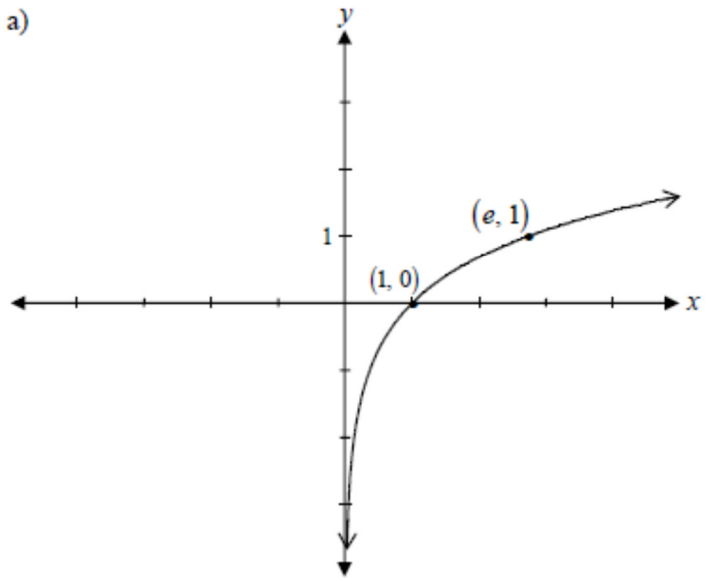
$y = 3$ ~~$y = -3$~~ — ½ mark for positive value of y; ½ mark for negative value of y and rejecting extraneous root

3 marks

Question 40 a) 2 marks b) 2 marks

Solutions

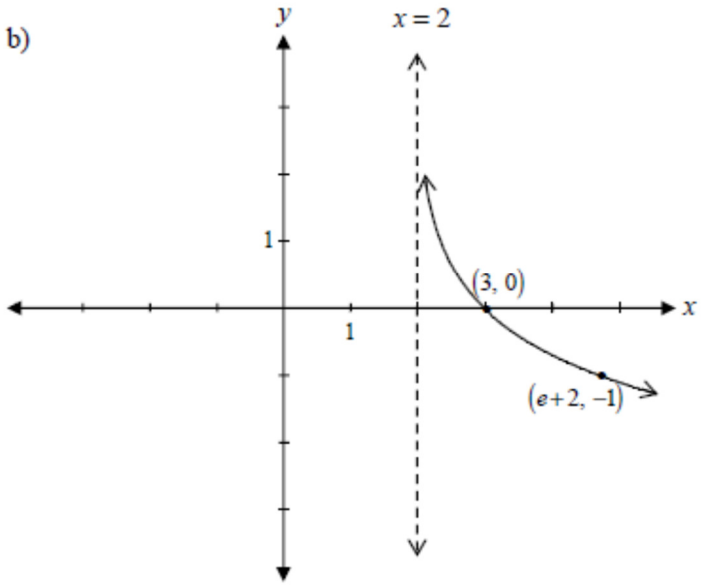
a)



½ mark for increasing logarithmic function
½ mark for x-intercept at $(1, 0)$
½ mark for consistent point on logarithmic function
½ mark for vertical asymptotic behaviour

2 marks

b)



1 mark for reflection in x-axis
1 mark for horizontal shift

2 marks