

### Expected Value

- An application of probability that involves the likelihood of a gain or a loss.
- Relevant in business, insurance, and many situations in your daily life.
- The concept of expected value can be used to determine whether or not to participate in games of chance.
- The expected value is an estimate of the average gain or loss when participating in a game of chance.

In general, if a game has:

- an expected value  $< 0$ , you can expect to lose \$ *negative*
- an expected value  $= 0$ , you can expect to break even
- an expected value  $> 0$ , you can expect to gain \$ *positive*

$$\text{Expected Value} = P(\text{Winning}) \times \text{Gain} - P(\text{Losing}) \times \text{Loss}$$

*amount paid*

#### Example 1

A game has 5 cards, one of which is the winning card. You pick a card:

- There is a 1 in 5 chance of winning \$4
- There is a 4 in 5 chance of winning \$0
- It costs \$1 to play each time

a) Calculate the expected value of the game.

$$\begin{aligned} \text{Expected Value} &= P(\text{Winning}) \times \text{Gain} - P(\text{Losing}) \times \text{Loss} \\ &= \frac{1}{5} (\$4) - \frac{4}{5} (\$1) \\ &= -\$0.20 \end{aligned}$$

*\$4 - \$1*

b) If you played this game 50 times, how much would you expect to win or lose?

$$\begin{aligned} &\text{EV} \times \# \text{ times played} \\ &= -\$0.20 \times 50 \\ &= -\$10.00 \end{aligned}$$

*← expected to lose \$0.20 (on average) each time*

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EM40S

Lesson 4

Expected Value

## Example 2

Consider the following game:

- There is a 1 in 10 chance of winning \$25
- There is a 2 in 10 chance of winning \$5
- There is a 7 in 10 chance of winning \$0
- It costs \$3 to play each time

gain  $\$25 - 3 = \$22$   
 $\$5 - 3 = \$2$

a) Calculate the expected value of the game.

$$EV = \frac{1}{10}(22) + \frac{2}{10}(2) - \frac{7}{10}(3)$$
  
$$= \$0.50$$

On average, you're expected to gain \$0.50 per time played.  
(based on multiple plays)

b) How much do you expect to win or lose, if you play this game 75 times?

$$\$0.50 \times 75 = \$37.50$$

## Example 3

A local sports team holds a raffle. Each raffle ticket costs \$2 and the raffle consists of 2500 tickets. One ticket will be drawn for the grand prize of a vacation package worth \$3000.

a) Determine your expected value if you buy 1 ticket.

$$EV = \left(\frac{1}{2500}\right)(\$2998) - \left(\frac{2499}{2500}\right)(\$2)$$
  
$$= -\$0.80$$

b) Determine your expected value if you buy 5 tickets.

$$-\$0.80 \times 5 = -\$4.80$$

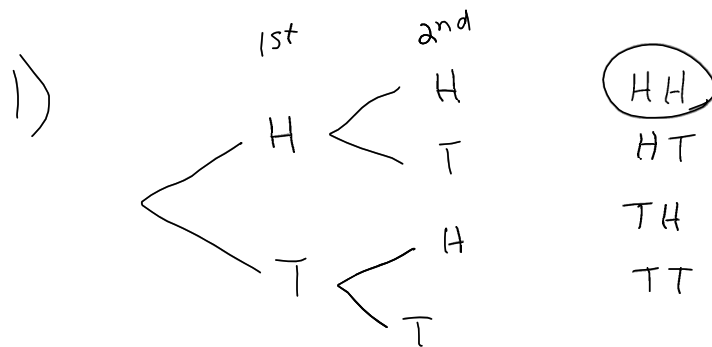
c) How much will the sports team gain or lose if it sells all 2500 tickets?

avg loss of \$0.80 → team gains \$0.80  
Total gain = \$0.80 × 2500  
= \$2000

or  $2500(\$2) - \$3000$   
 $\$2000$

Probability

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$$P(\text{win}) = \frac{1}{4}$$

$$P(\text{lose}) = \frac{3}{4}$$