PC40S Review – Exponents and Logarithms

Exponential Equations (Common Base)

Solve.

1. $27^x = 9^{2x-1}$
2. $4^{2x-1} = 64$
3. $6^{3x-6} = 1$
4. $2^{-x} = 128$
5. $5^{4-x} = \frac{1}{5}$
6. $32^{3x-2} = 64$
7. $2^{-2x} = 32$
8. $4^{8x} = \frac{1}{16}$
9. $3^{2x-1} + 1 = 2$
10. $3(5^{x+1}) = 15$

Answers:

1. 2
2. 2
3. 2
4. -7
5. 5
6. $\frac{16}{15}$
7. $-\frac{5}{2}$
8. $-\frac{1}{4}$
9. $\frac{1}{2}$
10. 0

Logarithms

Express in logarithmic form.

1. $3^2 = 9$
2. $6^3 = 216$
3. $4^5 = 1024$
4. $8^0 = 1$
5. $49^{\frac{1}{2}} = 7$
6. $5^{-2} = \frac{1}{25}$
7. $8^{\frac{2}{3}} = 4$
8. $9^{-1} = \frac{1}{9}$
9. $10^4 = 10\,000$
10. $a^b = c$

Answers:

1. $\log_3 9 = 2$
2. $\log_6 216 = 3$
3. $\log_4 1024 = 5$
4. $\log_8 1 = 0$
5. $\log_{49} 7 = \frac{1}{2}$
6. $\log_5 \frac{1}{25} = -2$
7. $\log_8 4 = \frac{2}{3}$
8. $\log_9 \frac{1}{9} = -1$
9. $\log_{10} 10\,000 = 4$
10. $\log_a c = b$

Express in exponential form.

1. $\log_5 5 = 1$
2. $\log_7 49 = 2$
3. $\log_3 729 = 6$
4. $\log_2 \frac{1}{16} = -4$
5. $\log_{10} 1 = 0$
6. $\log_{16} 4 = 0.5$
7. $\log_8 4 = \frac{2}{3}$
8. $\log_2 4096 = 12$
9. $\log 0.1 = -1$
10. $\log_3 \sqrt{3} = 0.5$

Answers:

1. $5^1 = 5$
2. $7^2 = 49$
3. $3^6 = 729$
4. $2^{-4} = \frac{1}{16}$
5. $10^0 = 1$
6. $16^{0.5} = 4$
7. $8^{\frac{2}{3}} = 4$
8. $2^{12} = 4096$
9. $10^{-1} = \frac{1}{10}$
10. $3^{0.5} = \sqrt{3}$

Evaluate

1. $\log_2 32$
2. $\log_3 27$
3. $\log 1000$
4. $\log_7 7$
5. $\log_9 1$
6. $\log_5 625$
7. $\log_2 2^9$
8. $\log_6 \frac{1}{36}$
9. $\log_2 0.25$
10. $\log 0.0001$
11. $\log_6 \sqrt{6}$
12. $\log_2 8\sqrt{2}$

Answers:

1. 5
2. 3
3. 3
4. 1
5. 0
6. 4
7. 9
8. -2
9. -2
10. -4
11. $\frac{1}{2}$
12. $\frac{7}{2}$

Evaluate using the Change of Base Theorem

1. $\log_{13} 6$
2. $\log_6 17$
3. $\log_3 \frac{1}{5}$
4. $\log_4 8$

Answers:

1. 0.699
2. 1.581
3. -1.465
4. 1.5

Laws of Logarithms

Evaluate.

1. $2\log_3 12 - 2\log_3 4$
2. $\log_4 6 + \log_4 \frac{64}{3} - \log_4 8$
3. $\log_3(9 \times 27 \times 81)$
4. $\frac{1}{2}\log_3 144 - \log_3 4 + \log_3 3.$
5. $7^{\log_7 3}$
6. $\log_5 \sqrt{175} - \log_5 \sqrt{7}$
7. $\ln \frac{1}{e^2} + \ln e^3$

Answers:

1. 2
2. 2
3. 9
4. 2
5. 3
6. 1
7. 1

Simplify, using laws of logarithms.

1. $\log_2 a + \log_2 b - \log_2 c$
2. $\log x^2 - 5\log y$
3. $\log A + \log \sqrt{B} - 3\log C$
4. $\log_7 \sqrt[3]{x} - \log_7 y^3 + 2\log_7 y$
5. $\ln(x+3) - \ln(2x+5) + 2\ln(x-1)$

Answers:

1. $\log_2 \frac{ab}{c}$
2. $\log \frac{x^2}{y^5}$
3. $\log \frac{A\sqrt{B}}{C^3}$
4. $\log_7 \frac{\sqrt[3]{x}}{y}$
5. $\ln\left(\frac{x^2+2x-3}{2x+5}\right)$

Logarithmic Equations

Solve.

1. $\log_2(x-2)+\log_2 x=\log_2 3$
2. $\log_2(x-2)+\log_2 x=3$
3. $\log_5(3x+1)+\log_5(x-3)=3$
4. $\log_9(x-5)=1-\log_9(x+3)$
5. $\log_2(x^2+8)-\log_2 6=\log_2 x$
6. $\log(2x+1)=1+\log(x-2)$
7. $\log_3(x-2)+\log_3 10-\log_3(x^2+3x-10)=0$
8. $(\log_3 x)^2=\log_3 x^2+3$
9. $\ln(x+1)+\ln(x-1)=\ln(x+5)$
10. $\ln(5x+8)=\ln(40-3x)$

Answers:

1. 3	2. 4
3. 8	4. 6
5. 2, 4	6. $\frac{21}{8}$
7. 5	8. $\frac{1}{3}, 27$
9. 3	10. 4

Exponential Equations (No Common Base)

Solve. Express your answers correct to 3 decimal places.

1. $2^{x+3}=17^x$
2. $17^{x+4}=196^{3x-2}$
3. $21^{2x+5}=278^{3x-7}$
4. $0.63^{x-4}=5^{2x}$
5. $7\times 2^x=5^{x-2}$
6. $485\times 5^{x+2}=12^{2x-1}$
7. $4+3e^{2x+1}=8$
8. $3^{2x-1}=2^{2-x}$
9. $e^{2x}=7$
10. $e^{4x}=5^{3x+1}$

Answers:

1. 0.972	2. 1.684
3. 5.060	4. 0.502
5. 5.637	6. 3.358
7. −0.356	8. 0.860
9. 0.973	10. −1.943

Applications

1. Melissa has recently inherited $15 000 that she wants to deposit into a savings account for 10 years. She has determined that her two best bets are an account that compounds annually at a rate of 3.95% and an account that compounds continuously at an annual rate of 3.85%. Which account would pay Melissa more interest?

2. Bill has come upon a 37 gram sample of iodine-131. He isolates the sample and waits for 2 weeks. After this time period, only 11 grams of iodine-131 remains. What is the half-life of this isotope?

3. Katherine is working in a lab, testing bacteria populations. Starting with a population of 870 bacteria, she notices that the population doubles every 22 minutes. Determine the time it would take for the population to reach 7 500 bacteria. Use the equation $A = Pe^{rt}$, where *A* is the final amount, *P* is the original amount, *r* is the rate of growth, and *t* is the time in minutes.

4. The number of fruit flies in an experimental population after t hours is given by $Q(t) = 20e^{0.03t}, t \geq 0$.

   a) How large is the population of fruit flies after 72 hours?

   b) Find the initial number of fruit flies in the population.

5. An investment earns interest at a rate of 4% compounded quarterly ($n = 4$). Determine how the number of years it will take for the investment to double. Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ where *r* is the annual rate of increase as a percent, and *t* is the time in years.

6. In April 1994, the population of a small town in Manitoba was estimated at 2 500 people. The population can be represented by the equation $A = Pe^{rt}$, where *A* is the final amount, *P* is the original amount, *r* is the annual rate of increase as a percent, and *t* is the time in years. Determine the annual rate of increase if there were 3 900 people in April 1999. Express your answer correct to three decimal places.

Answers:

1. $3.95\%: \$22\,097.15, 3.85\%: \$22\,044.21$
2. 1.14 weeks, 8 days
3. 68.386 minutes
4. a) 173 fruit flies b) 20
5. 17.415 years
6. 8.894%