

## Pre-Calculus 11 Equivalent Rational Expressions

When the numerator and denominator of a fraction are integers, the fraction is a rational number.

Example:  $\frac{3}{4}$ , or  $\frac{-2}{5}$

When the numerator and denominator of a fraction are polynomials, the fraction is a rational expression.

For example,  $\frac{3x-2}{4x-3}$ ,  $\frac{x^2-4}{x^2-x-12}$ , and  $\frac{4x}{x^2+2}$  are all rational expressions.

**Rational Expression** – any algebraic expression that can be written as the quotient of two polynomials.

Rational expressions cannot contain roots of variables or variables in exponents.

For example,  $\frac{3x-2}{\sqrt{x+2}}$  and  $\frac{2+2^x}{x}$  are not rational expressions.

### Evaluating Rational Expressions

To evaluate a rational expression, simply fill in the number given for each variable.

For example if  $x = 2$  and  $y = -2$  evaluate:  $\frac{2x-3y^2}{3x+2y}$

$$\frac{2(2) - 3(-2)^2}{3(2) + 2(-2)}$$

$$\frac{-8}{-4}$$

### Non-permissible Values

Rational expressions are not defined for values of the variable that make the denominator 0 (you cannot divide by 0). These values are called non-permissible values or restrictions.

**Example:**  $\frac{x+5}{x-3}$  is not defined for  $x = 3$ . So,  $x = 3$  is a non-permissible value.  
*denom  $x-3 = 0$   $x = 3$*

Find the non-permissible values for the following rational expressions.

- To find the non-permissible values, set the denominator equal to 0 and solve for x.

a)  $\frac{5x}{x^2-9}$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

or diff of squares

$$(x-3)(x+3) = 0$$

$$x = 3 \quad x = -3$$

b)  $\frac{3x+2}{x^2-8x+16}$

$$x^2 - 8x + 16 = 0$$

$$(x-4)(x-4) = 0$$

$$x = 4$$

**Simplifying Rational Expressions**

To simplify a rational expression, we use the same process as simplifying a fraction.

$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

$$\frac{20xy}{8x^2} = \frac{20xy \div 2x}{8x^2 \div 2x} = \frac{10y}{4x}$$

$$\frac{\cancel{3(x+2)}}{(x+2)(x-4)} = \frac{3}{x-4}$$

npv's  $x = -2, 4$

The rational expression has  $x = 0$  as a non-permissible value.

In order to simplify a rational expression, the following must be true:

- The rational expression must be in reduced form. This means that it must be factored first and reduced if applicable.
- The restrictions (Non-Permissible Values) for the variable in the denominator must be stated as part of the final answer. The denominator cannot equal zero. Always use the original question to state restrictions and not just the simplified answer.

**Examples: Simplify and State Restrictions**

1.  $\frac{-25x^4y^6}{35x^2y^9} =$

GCF  $5x^2y^6$

$$\frac{-5x^2}{7y^3}$$

npv's  $x, y = 0$   
restrictions  $x, y \neq 0$

$x, y \neq 0$

2.  $\frac{4x-12}{6x-24} =$

\*Factor first !!

$$\frac{4(x-3)}{6(x-4)}$$

$x \neq 4$

$$\frac{2(x-3)}{3(x-4)}$$

3.  $\frac{x^2-x-12}{x-4}$

$\frac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}}$

$x+3$

$x \neq 4$

*P - 12*  
*S - 1*  
*F - 4, 3*

4.  $\frac{2x^2-5x-3}{x^2-9}$

$\frac{(2x+1)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$

$\frac{2x+1}{x+3}$

$x \neq \pm 3$

*P - 6*  
*S - 5*  
*F - 6, 1, 1*

5.  $\frac{16-x^2}{2x-8}$

$\frac{(4+x)(4-x)}{2(x-4)}$

$-(x+4)(\cancel{x-4})$

$\frac{-(x+4)}{2}$

$x \neq 4$

$-x+4$   
 $-(x-4)$

GCF -1

**Assignment:** Pg. 527; #4, 5, 6, 7a,c 8a, b, 10a, 11a, 12