Pre-Calculus 12 Double Angle Identities

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\cos 2\alpha = 1 - 2\sin^2\alpha$
$\cos 2\alpha = 2\cos^2\alpha - 1$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

Ex. 1) Write $2\sin 5\cos 5$ in terms of a single trigonometric function.

↳ follows the pattern $2\sin\alpha\cos\alpha$
RHS

$\alpha = 5$

LHS
$\sin 2\alpha$
$\sin 2(5)$
$\sin 10$

Ex. 2) Write the following in terms of only one circular function.

a) $\cos^2 4x - \sin^2 4x$

follows pattern $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\alpha = 4x$

$\boxed{\cos 2(4x)} = \cos^2 4x - \sin^2 4x$

$\boxed{\cos 8x}$

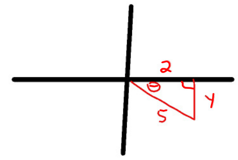
b) $4\sin x\cos x$

$\boxed{2\sin 2\alpha} = 2(2\sin\alpha\cos\alpha)$

$\therefore 2\sin 2x$

Ex. 3) Given angle θ in standard position with its terminal arm in Q4 and $\cos\theta = \frac{2}{5}$, determine the exact value of:

a) $\sin 2\theta$



$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{\sqrt{21}}{5}\right)\left(\frac{2}{5}\right)$
$= \frac{-4\sqrt{21}}{25}$

$2^2 + y^2 = 5^2$
$y^2 = 21$
$y = \pm\sqrt{21}$
in QIV
$\therefore y = -\sqrt{21}$
$\sin\theta = -\frac{\sqrt{21}}{5}$

b) $\cos 2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$
$= 2\left(\frac{2}{5}\right)^2 - 1$
$= 2\left(\frac{4}{25}\right) - 1$
$= \frac{8}{25} - \frac{25}{25}$
$= \frac{-17}{25}$

Ex. 4) On the unit circle, the co-ordinates of the circular point $p(\theta)$ are $(0.588, 0.809)$. Find the co-ordinates of $P(2\theta)$.

$\cos\theta$ $\sin\theta$

$\hookrightarrow P(2\theta) = (\cos 2\theta, \sin 2\theta)$

$\cos 2\theta = 1 - 2\sin^2\theta$
$= 1 - 2(0.809)^2$
$= -0.309$

$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2(0.809)(0.588)$
$= 0.951$

$P(2\theta) = (-0.309, 0.951)$

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\# 1c, e
2c
4c
5a
11a, b
20c, d

Ex. 5) Prove $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

$$\frac{\frac{2\sin\theta}{\cos\theta}}{\sec^2\theta}$$

$$\frac{\frac{2\sin\theta}{\cancel{\cos\theta}}}{\frac{1}{\cos^{\cancel{2}}\theta}}$$

$2\sin\theta\cos\theta$

$\sin 2\theta = RHS$ ✓

$$\frac{2\sin\theta}{\cos\theta} \div \frac{1}{\cos^2\theta}$$

$$\frac{2\sin\theta}{\cancel{\cos\theta}} \cdot \cos^{\cancel{2}}\theta$$

Ex. 6) Prove the identity $\tan\theta + \cot\theta = 2\csc 2\theta$ for all permissible values of θ.

LCD

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \quad \Bigg| \quad \frac{2}{\sin 2\theta}$$

$$\frac{\cancel{\sin^2\theta + \cos^2\theta}}{\sin\theta\cos\theta} \quad \Bigg| \quad \frac{\cancel{2}}{\cancel{2}\sin\theta\cos\theta}$$

$$\frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$ ✓

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\# 1, c, e, 2c,
4c, 5a, c, d
11a, b, 16,
20c, d

pg. 314
\# 10 a, c