Derivatives of Logarithmic Fcns

ex Differentiate

a) $\ln 3x$

$\frac{d}{dx}\ln 3x = \frac{1}{3x}(3)$

$= \frac{1}{x}$

b) $\ln x^4$

$\frac{d}{dx}\ln x^4 = \frac{1}{x^4}(4x^3)$

$= \frac{4}{x}$

c) $\ln(2x+3)$

$\frac{d}{dx}\ln(2x+3) = \frac{1}{2x+3}(2)$

$= \frac{2}{2x+3}$

d) $\ln(2x^2+3x-5)$

$\frac{d}{dx}\ln(2x^2+3x-5) = \frac{1}{2x^2+3x-5}(4x+3)$

$= \frac{4x+3}{2x^2+3x-5}$

ex.2 Differentiate

$f(x) = \log_{10}(2+\sin x)$

$f'(x) = \frac{1}{(2+\sin x)\ln 10}\cdot \cos x$

$= \frac{\cos x}{(2+\sin x)\ln 10}$

ex.3 Find $\frac{dy}{dx}$ if $y = \log_a a^{\sin x}$

$\frac{dy}{dx} = \frac{1}{a^{\sin x}\ \ln a}\cdot a^{\sin x}\cdot \ln a \cdot \cos x$

$= \cos x$

* Easier to simplify first.

$y = \log_a a^{\sin x} \rightarrow a^y = a^{\sin x}$

$y = \sin x \log_a a$

$y = \sin x$

is

$y = \sin x$

$\frac{dy}{dx} = \cos x$

Proof

$\frac{d}{dx}\ln x = \frac{1}{x}$

$y = \ln x$

$e^y = x$

implicit differentiation

$\frac{d}{dx}e^y = \frac{d}{dx}x$

$e^y\cdot\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y}$

$\frac{dy}{dx} = \frac{1}{x}$

Proof

$$\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$$

$\log_2 8 = 3$

$\frac{\log 8}{\log 2}$

change of base

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\frac{d}{dx}\log_a x = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right)$$

$$= \frac{1}{\ln a}\frac{d}{dx}\ln x$$

$$= \frac{1}{\ln a}\cdot\frac{1}{x}$$

$$= \frac{1}{x\ln a}$$

It Follows

$$\frac{d}{dx}\log_a u = \frac{1}{u\ln a}\cdot\frac{du}{dx}$$

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