Curve Sketching

Steps

① Find the domain.

② Find all intercepts

③ Use $f'(x)$ to determine where the graph is increasing/decreasing

ie Use the First Derivative Test w/ critical values

④ Use $f''(x)$ to find intervals of concavity

ie Second Derivative Test

⑤ Sketch the curve.

Point of Inflection

$f(c)$ is defined

$f''(x)$ changes sign as x increases through $x = c$

Second Derivative Test

Solve $f''(x) = 0$ to determine

If $f''(x) > 0$, then $f(x)$ is concave up over that interval

If $f''(x) < 0$, then $f(x)$ is concave down over that interval

ex.1 Sketch $f(x) = 3x^5 - 5x^3$

① $D: x \in \mathbb{R}$

② intercepts

x-int

$3x^5 - 5x^3 = 0$

triple root → $x^3(3x^2 - 5) = 0$

$x^3 = 0$ $3x^2 - 5 = 0$

graph flattens $\boxed{x = 0}$ $x^2 = \frac{5}{3}$

$x = \pm\sqrt{\frac{5}{3}}$

$\boxed{x = \pm 1.29}$

y-int

$f(0) = 3(0)^5 - 5(0)^3$

$\boxed{y = 0}$

③ $f'(x)$

$f'(x) = 15x^4 - 15x^2$

$0 = 15x^4 - 15x^2$

$0 = 15x^2(x^2 - 1)$

$15x^2 = 0$ $x^2 - 1 = 0$

$x = 0$ $x = \pm 1$

max   min

$+ \quad - \quad - \quad +$

$-1 \quad 0 \quad 1$

double root

original f(x)

$f(-1) = 3(-1)^5 - 5(-1)^3$

$= 2$ ← max $(-1, 2)$

$f(1) = 3(1)^5 - 5(1)^3$

$= -2$ ← min $(1, -2)$

④ $f''(x) = 60x^3 - 30x$

$0 = 60x^3 - 30x$

$0 = 30x(2x^2 - 1)$

$x = 0$ $x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

$x = \pm 0.7$

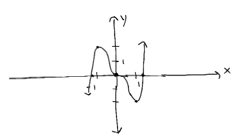
c.d   c.u   c.d   c.u.

$- \quad + \quad - \quad +$

$-0.7 \quad 0 \quad 0.7$

c.d → concave down

c.u → concave up



ex.2 Graph the fcn $f(x) = 2x^3 + 3x^2 - 5$ whose factored form is $f(x) = (x-1)(2x^2 + 5x + 5)$

① $D: x \in \mathbb{R}$

② x-int

$0 = (x-1)(2x^2 + 5x + 5)$

$x = 1$ $x = \frac{-5 \pm \sqrt{25 - 4(2)(5)}}{2(2)}$

$x = \frac{-5 \pm \sqrt{-15}}{4}$

ϕ

y-int

$f(0) = -5$

③ $f'(x) = 6x^2 + 6x$

$0 = 6x^2 + 6x$

$0 = 6x(x + 1)$

$x = 0$ $x = -1$

inc   dec   inc

max   min

$+ \quad - \quad +$

$-1 \quad 0$

max $f(-1) = 2(-1)^3 + 3(-1)^2 - 5$

$= -4$

min $f(0) = -5$

④ $f''(x) = 12x + 6$

$0 = 12x + 6$

$-6 = 12x$

$-\frac{1}{2} = x$

c.d.   c.u.

$- \quad +$

$-\frac{1}{2}$