Continuity

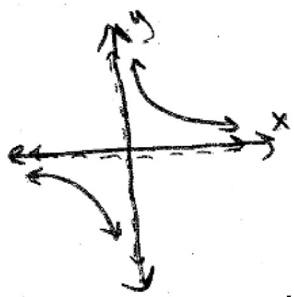
A fcn, f, is continuous at a point $(a, f(a))$ if $\lim_{x \to a} f(x) = f(a)$

This implies:

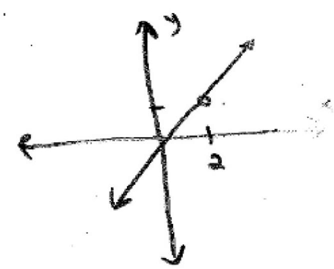
1) the value of $f(x)$ at $x = a$ is defined
2) the $\lim_{x \to a} f(x)$ exists
3) these 2 values are equal

A function, f, is said to be continuous on an open interval (a, b) if it is continuous at each point of (a, b)

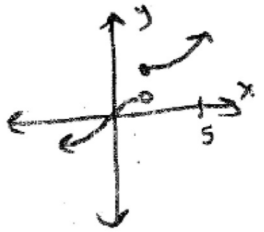
ex



infinite discontinuity



point discontinuity



jump discontinuity

ex Is $f(x) = x^2 - 3x - 4$ continuous at $x = 2$?

① $f(2) = 2^2 - 3(2) - 4$
$= -6$

② $\lim_{x \to 2} x^2 - 3x - 4$
$2^2 - 3(2) - 4$
-6

$\therefore \lim_{x \to 2} f(x)$ exists

③ $f(2) = \lim_{x \to 2} f(x)$ ✓ or $\lim_{x \to 2} f(x) = f(2)$

$\therefore f(x)$ is continuous at $x = 2$

ex 3 Given $f(x)$ is $f(x)$ continuous at 3?

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & x \neq 3 \\ 4 & x = 3 \end{cases}$$

Algebraically

① $f(3) = 4$

② $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

$\frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$

$3 + 3$

6

③ $f(3) \neq \lim_{x \to 3} f(x)$

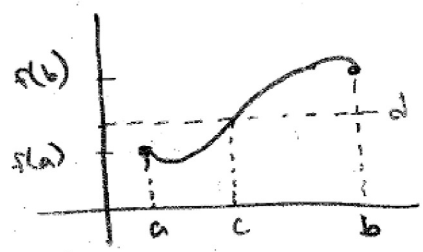
$\lim_{x \to 3} f(x) \neq f(3)$

$\therefore f(x)$ is not continuous at 3

The Intermediate Value Thm

A fcn is said to have the intermediate value property if it never takes on two values w/o taking on all the values in between.

A fcn $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$.



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\# 1, 3, 11-13
19, 21, 23
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worksheet
\# 1-8
10, 12,
13, 14

hand in