Combining Functions

January 2014
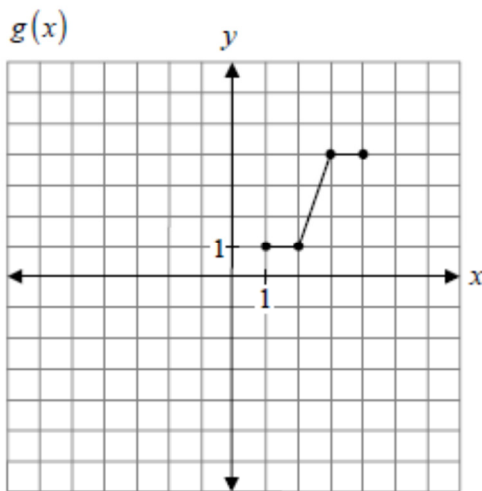
Question 10   2 marks

Given the graphs of $f(x)$ and $(f - g)(x)$, sketch the graph of $g(x)$.

Solution

$g(x)$

1 mark for subtraction of $f(x) - (f - g)(x)$
1 mark for shape representing the operation given

2 marks

Question 15   1 mark

If $f(x) = x^3$ and $g(x) = 2x - 3$, what is the value of $\left(\frac{f}{g}\right)(-1)$?

Solution

$f(-1) = (-1)^3$
$\quad = -1$

½ mark for substituting into $f(x)$ and $g(x)$

$g(-1) = 2(-1) - 3$
$\quad = -5$

$\left(\frac{f}{g}\right)(-1) = \frac{-1}{-5}$

$\quad = \frac{1}{5}$

½ mark for evaluating $\left(\frac{f}{g}\right)(-1)$

1 mark

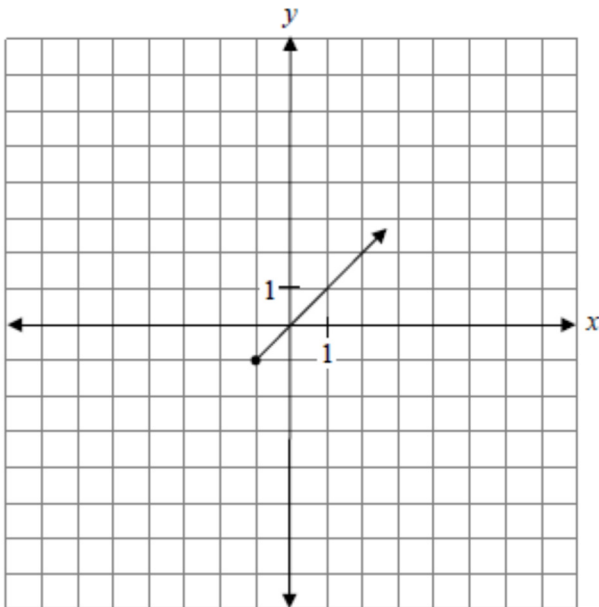## Question 29   3 marks

Given $f(x) = x^2 - 1$ and $g(x) = \sqrt{x + 1}$, sketch the graph of $y = f(g(x))$ and state its domain.

Solution

Method 2

x	$g(x)$	$f(g(x))$
−2		
−1	0	−1
0	1	0
1		1
2		2
3	2	3

1 mark for table of values

1 mark for graph of composite function

Domain of $f(g(x)) : [-1, \infty)$ or $\{x \mid x \geq -1, x \in \mathbb{R}\}$

1 mark for restricted domain

3 marks

June 2013

Question 9   1 mark

Given that $f(x) = \{(1, 3), (2, 5), (3, 4), (4, 2)\}$, find $f(f(3))$.

Solution

$$f(f(3)) = f(4)$$
$$= 2$$

½ mark for $f(3) = 4$
½ mark for $f(4) = 2$

1 mark
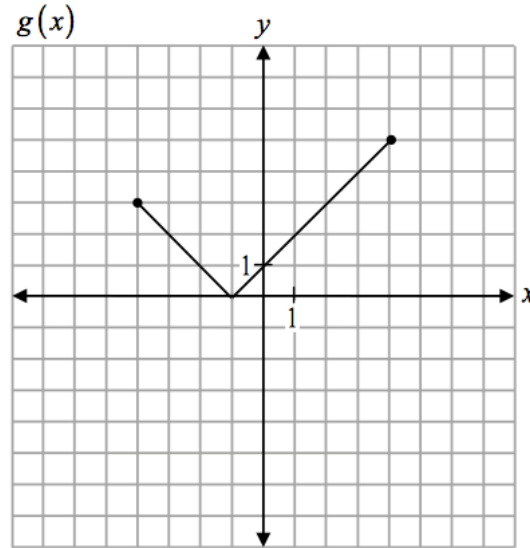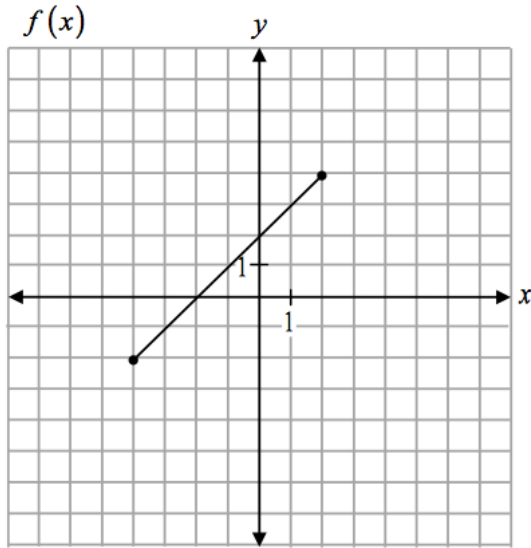
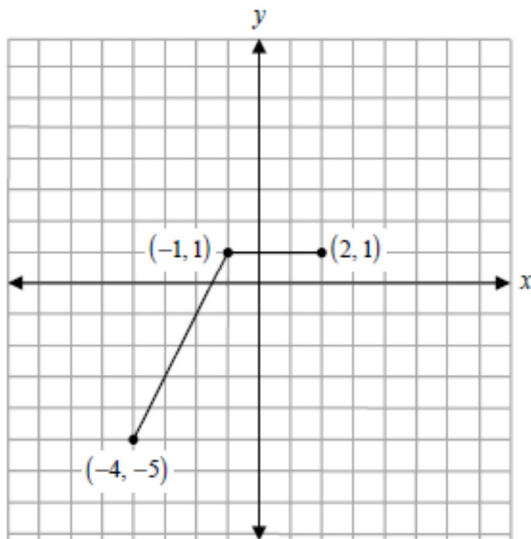## Question 10   2 marks

Given the graph of $f(x)$ and $g(x)$ below,

$f(x)$

$g(x)$

Sketch the graph of $y = f(x) - g(x)$

Solution

x	$f(x)$	$g(x)$	$f(x) - g(x)$
-4	-2	3	-5
-2	0	1	-1
-1	1	0	1
0	2	1	1
2	4	3	1

$(-1, 1)$ $(2, 1)$

$(-4, -5)$

1 mark for subtraction of $f(x) - g(x)$
1 mark for restricted domain

2 marks

Question 26   2 marks

Given $f(x) = 3$ and $g(x) = x + 2$, determine the domain and range of $h(x) = \frac{f(x)}{g(x)}$.

Solution

Domain: $\{x | x \in \mathbb{R}, x \neq -2\}$

1 mark for domain

Range: $\{y | y \in \mathbb{R}, y \neq 0\}$

1 mark for range

2 marks

Question 41   2 marks

Given $f(x) = \sqrt{x - 2}$ and $g(x) = 3x$, write the equation for $h(x) = f(g(x))$.

What are the restrictions on the domain of $h(x)$?
Explain your reasoning.

Solution

$h(x) = \sqrt{3x - 2}$

1 mark for $h(x) = f(g(x))$

$3x - 2 \geq 0$
$3x \geq 2$
$x \geq \dfrac{2}{3}$

½ mark for identifying restriction

Since we cannot find a square root of a negative number, there is a restriction

½ mark for explanation

on the domain, $x \geq \dfrac{2}{3}$.

2 marks

Question 45   2 marks

Given $f(x) = x - 1$ and $g(x) = x^2$, write the equation of $y = f(g(x))$ and sketch the graph.

Solution
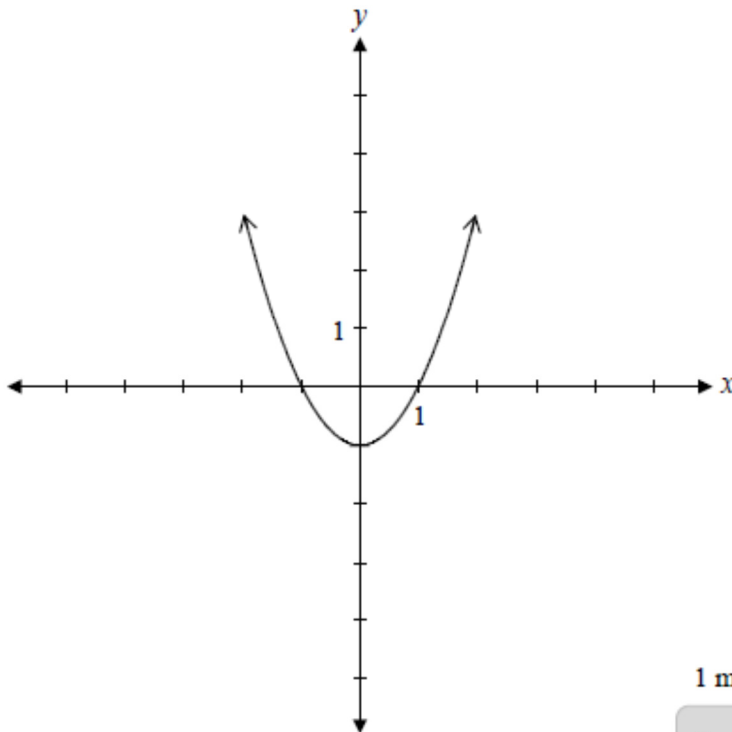
$$f(g(x)) = x^2 - 1$$ 1 mark for composition

or

$$y = x^2 - 1$$

1 mark for consistent graph

2 marks