Pre-Calculus 12 Combining Functions Algebraically

Recall:

$h(x) = f(x) + g(x)$ same as $h(x) = (f + g)(x)$

$h(x) = f(x) - g(x)$

$h(x) = f(x) \cdot g(x)$

$h(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

Example 1: Sum of Functions

a) Given $f(x) = -x - 5$ and $g(x) = (x + 3)^2$, write an explicit equation for $h(x) = f(x) + g(x)$.

$h(x) = f(x) + g(x)$

$= -x - 5 + (x+3)^2$ sub in expressions for $f(x)$ and $g(x)$

$= -x - 5 + x^2 + 6x + 9$

$= x^2 + 5x + 4$

b) Determine the domain of $h(x)$.

domain of $f(x)$ is $x \in \mathbb{R}$

domain of $g(x)$ is $x \in \mathbb{R}$

$\therefore$ domain of $h(x)$ is $x \in \mathbb{R}$

c) Evaluate $h(-2)$. (x is replaced by -2)

$h(x) = x^2 + 5x + 4$

$h(-2) = (-2)^2 + 5(-2) + 4$

$= -2$

Example 2: Difference of Functions

a) Given $f(x) = x^2$ and $g(x) = 2x + 1$, write an explicit equation for $h(x) = f(x) - g(x)$.

$h(x) = f(x) - g(x)$

$= x^2 - (2x+1)$ ← $g(x)$ must be in brackets

$= x^2 - 2x - 1$

b) Evaluate $h(3)$.

replace x with 3

$h(3) = 3^2 - 2(3) - 1$

$= 2$

Example 3: Product of Functions

Given $f(x) = 2x - 1$ and $g(x) = x^2 + 4$, write an explicit equation for $h(x) = f(x) \cdot g(x)$.

$h(x) = f(x) \cdot g(x)$

$= (2x-1)(x^2+4)$ ← each function is bracketed

$= 2x^3 + 8x - x^2 - 4$

or

$= 2x^3 - x^2 + 8x - 4$

FOIL

if worth 1 mark, stop

Example 4: Quotient of Functions

a) Given $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$, write an explicit equation for $h(x) = \frac{f(x)}{g(x)}$.

parabola line

$h(x) = \frac{f(x)}{g(x)}$

$= \frac{x^2 + x - 6}{2x + 6}$

$= \frac{(x+3)(x-2)}{2(x+3)}$ ← factor numerator and denominator

$= \frac{x-2}{2}$

$x \neq -3$ ← remember restrictions

b) State the domain of $h(x)$.

domain of $f(x)$ is $x \in \mathbb{R}$

domain of $g(x)$ is $x \in \mathbb{R}$

but for $h(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

$2x + 6 \neq 0$

$2x = -6$

$\therefore x \neq -3$

c) Write an explicit equation for $h(x) = \frac{g(x)}{f(x)}$

$h(x) = \frac{2x+6}{x^2 + x - 6}$

$= \frac{2(x+3)}{(x+3)(x-2)}$

$= \frac{2}{x-2}$

$f(x) \neq 0$

$(x+3)(x-2) \neq 0$

$x \neq -3 \quad x \neq 2$

Example 5: Given $p(x) = x^2 - 9$, write the explicit equations for two functions $f(x)$ and $g(x)$ so that $p(x) = f(x) \cdot g(x)$.

$p(x) = x^2 - 9$

$= (x-3)(x+3)$ ← factor as a diff of squares

$\therefore f(x) = x - 3 \quad g(x) = x + 3$

pg. 483 # 1 b, c, 2 b, 3 a, c, 10 a

pg. 496 # 1 a, b, 4 a 5 a, 8 a

determine $h(x) = (f \cdot g)(x)$ and $h(x) = \left(\frac{f}{g}\right)(x)$