Lesson 1 Combining Functions Algebraically

Sum, Difference, Product, and Quotient of Functions

$(f+g)(x) = f(x) + g(x)$ $\quad$ $(f-g)(x) = f(x) - g(x)$

$(f \cdot g)(x) = f(x) \cdot g(x)$ $\quad$ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

Example 1

Write an expression for each combination of functions, given the functions $f, g, h,$ and k are defined as follows:

$f(x) = 2x - 5, \quad g(x) = (x+2)^2, \quad h(x) = x^2 - 1, \quad k(x) = x + 1$

a.) $f(x) + g(x)$

Sub in expressions for f(x) and g(x)

$2x - 5 + (x+2)^2$

$2x - 5 + x^2 + 4x + 4$

$x^2 + 6x - 1$

b.) $h(x) - k(x)$

$x^2 - 1 - (x+1)$

$x^2 - 1 - x - 1$

$x^2 - x - 2$

c.) $(f \cdot h)(2) + (g \cdot k)(-1)$

same as → $f(2) \cdot h(2) + g(-1) \cdot k(-1)$

$(-1)(3) + (1)(0)$

-3

Evaluate f(2), h(2), g(-1) and k(-1) to sub in

$f(x) = 2x - 5$

$f(2) = 2(2) - 5$

$= -1$

$h(x) = x^2 - 1$

$h(2) = 2^2 - 1$

$= 3$

and so on...

Example 2

Write an expression for each combination of functions, given the functions $f, g, h,$ and k are defined as follows:

$f(x) = 2x + 6, \quad g(x) = x^3, \quad h(x) = x^2 + x - 6, \quad k(x) = 2$

$k(1) = 2$

a.) $f(x) \cdot g(x)$

$(2x + 6)(x^3)$

or

$2x^4 + 6x^3$

b.) $\frac{k(1)}{g(2)}$

$\frac{2}{8}$

$\frac{1}{4}$

c.) $\left(\frac{hk}{f}\right)(x)$ $\quad \frac{h(x) \cdot k(x)}{f(x)}$

$\frac{(x^2 + x - 6)(2)}{2x + 6}$

factor and simplify

$\frac{2(x - 2)(x + 3)}{2(x + 3)}$

$x - 2$ $\quad x \neq -3$ ← remember restrictions

Example 3

a) Given $f(x) = x^2 - 9$ and $g(x) = x - 3$, write an explicit equation for $h(x) = \frac{f(x)}{g(x)}$.

$h(x) = \frac{x^2 - 9}{x - 3}$

$= \frac{(x - 3)(x + 3)}{x - 3}$

$= x + 3$

b) State the domain of $h(x)$.

$x \neq 3$

c) Write an equation for $h(x) = \frac{g(x)}{f(x)}$

$h(x) = \frac{x - 3}{x^2 - 9}$

$= \frac{x - 3}{(x + 3)(x - 3)}$ $\quad x \neq \pm 3$

$= \frac{1}{x + 3}$

Example 4

Given $p(x) = x^2 - 25$, write the equations for two functions $f(x)$ and $g(x)$ so that $p(x) = f(x) \cdot g(x)$.

factors of $p(x)$

$p(x) = x^2 - 25$

$= (x - 5)(x + 5)$ ← factor

$\therefore f(x) = x - 5$

$g(x) = x + 5$

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1, 2
3 d, g, h
4 b, d