Pre-Calculus 12 Enriched
Combining Functions/Trigonometry Extension

1. Complete the chart.

Point on $y = f(x)$	Point on $y = g(x)$	Point on $y = (f \circ g)(x)$
(–19, 29)	(–3, –19)	
	(2, 1)	(2, –1)
(–7, 11)	(0,)	
(, 17)	(,–11)	(–1,)
(3,)		$\left(\frac{5}{2}, -4\right)$

2. If $f(x) = ax^2 + ax - 1$, $g(x) = bx^2 + 3x - 5$ and $(f - g)(x) = 4x^2 - ax + c$, determine the values of a, b, and c.

3. Given the functions $f(x) = ax - 1$ and $g(x) = bx^2 + 3x + 5$, determine the value of a and b if $(g \cdot f)(x) = -48x^3 + 26x^2 + 27x - 5$.

4. Given the function $h(x) = \sqrt{4 - x} + 2\sqrt{x} - 4$,

   a. state the domain of the $h(x)$.
   b. determine the x-intercepts of the function.
   c. Using the graphs of $f(x) = \sqrt{4 - x}$ and $g(x) = 2\sqrt{x} - 4$, sketch the graph of $y = h(x)$.

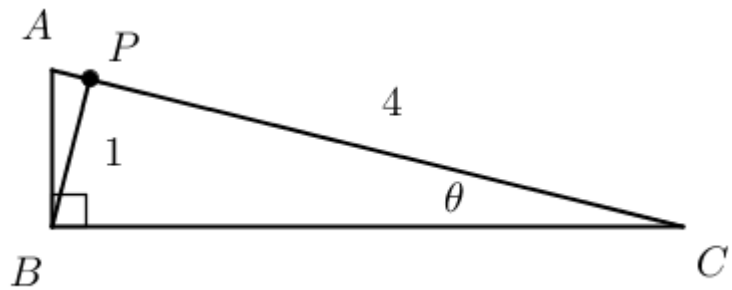
5. **Definition:** A function, $f(x)$, is an **even function** if $f(-x) = f(x)$ for all values of x. A function, $f(x)$, is an **odd function** if $f(-x) = -f(x)$ for all values of x.
   Prove that

   a. The sum of two even functions is an even function.
   b. The sum of two odd functions is an odd function.

6. Given $f(x) = x^3 - 1$ and $g(x) = x^3 + 1$, prove that $\frac{f(x)}{g(x+1)} = \frac{x-1}{x+2}$ for $x > -2$.

7a.) Prove that $tan(\theta) + cot(\theta) = 2csc(2\theta)$

b.) In the right-angled triangle ABC, $AC = 4$, $BP = 1$, and $BP \perp AC$. Determine a value for angle $\theta = \angle ACB$.



8. If $\sqrt{6}sin(\theta) + \sqrt{2}cos(\theta) = 2$ for $0 \leq \theta \leq 2\pi$, determine cos(2θ).

9. Prove the identity for all permissible values.

$$\frac{cos^3(\theta)+sin^3(\theta)}{sin(\theta)+cos(\theta)} = 1 - sin(\theta)cos(\theta)$$

10. Show that $(xsin(\theta) - ycos(\theta))^2 + (xcos(\theta) + ysin(\theta))^2 = x^2 + y^2$.