Pre-Calculus 12 Combinations

Combination... just select

Permutation... select and arrange

A ***combination*** is a selection of objects where order **DOES NOT** matter.

Ex. 1) The letters P, Q, R and S can be arranged into 3-letter combinations:

PQR	PRS	PQS	QRS	4 combinations
QRP	RSP	etc…	etc…	
RPQ	SPR			
PRQ	PSR			**24 permutations**
QPR	RPS			
RQP	SRP			

${}_4C_3$ choose 3

${}_4P_3$ choose and arrange 3

When order matters (permutations) there are ${}_4P_3$, or 24, ways to choose 3 letters from 4 letters. Then, there are 3!, or 6, ways to choose the same 3 letters.

So, the number of combinations is: $\frac{24}{3!} = 4$

Combinations of Different Objects

The number of combinations of n distinct objects taken r at a time is:

$$nCr = \frac{n!}{(n-r)!\,r!}, n \geq r$$

Note:

- ${}_nC_r$ can also be written as $\binom{n}{r}$ "n choose r", also C(n,r)
- ${}_nP_r$ is the number of ways to choose r objects from n
- r! is r objects can be arranged in r ways.

Key on calc

${}_4C_3$ $\binom{4}{3}$ $\binom{4}{3}$ C(4, 3)

different notations

Ex. 2) How many combinations are possible in Lotto 6/49?

↳ combination order doesn't matter

${}_{49}C_6$: 13 983 816 different combinations

Ex. 3) Lotto Max is a Canadian lottery where a player chooses 7 numbers from 1 to 49. To win the jackpot, all 7 numbers must match. Determine the probability that you will win Lotto Max.

$_{49}C_7 = 85\ 900\ 584$ combinations

$P(\text{win}) = \frac{1}{85\ 900\ 584}$ $\left(\frac{\#\text{ of successful ways}}{\text{total possible ways}}\right)$

1.16×10^{-8}

or $1.16 \times 10^{-6}\ \%$

Ex. 4) A local arena has 10 applicants interested in working in the snack bar.

a) How many ways can 4 applicants be chosen?

$_{10}C_4 = 210$ ways

b) How many ways can 6 applicants be chosen?

$_{10}C_6 = 210$ ways

$_nC_r = \frac{n!}{(n-r)!\,r!}$ w/ formula

$_{10}C_4 = \frac{10!}{(10-4)!\,4!} = \frac{10!}{6!\,4!}$

$_{10}C_6 = \frac{10!}{(10-6)!\,6!} = \frac{10!}{4!\,6!}$

equal

$\therefore\ _{10}C_4 = {}_{10}C_6$

$\therefore$ in general $_nC_r = {}_nC_{n-r}$

ie $_{15}C_8 = {}_{15}C_7$

$_{11}C_3 = {}_{11}C_8$

Ex. 5) Solve for n: $_nC_2 = 10$

$_nC_2 = 10$

$\frac{n!}{(n-2)!\,2!} = 10$

$\frac{n!}{(n-2)!} = 20$

$\frac{n(n-1)(n-2)!}{(n-2)!} = 20$

$n(n-1) = 20$

$n^2 - n - 20 = 0$

$(n-5)(n+4) = 0$

$n = 5$ $\quad n = -4$ rej

$n \geq 2$ because $r = 2$, can't select less than 2 objects

Ex. 6) In how many ways can a committee of 7 people be selected from 9 girls and 3 boys if exactly 2 boys must be on the committee?

total

exactly 2 boys so must chose 5 girls

$_3C_2 \cdot {}_9C_5 = 378$ ways

7 ppl on committee

Ex. 7) A new store must have 3 cashiers and 4 clerks. There are 7 applicants for cashier and 8 applicants for clerk. How many ways can 7 employees be chosen?

cashiers clerks

$_7C_3 \cdot {}_8C_4 = 2450$ ways

Case Examples

Ex. 8) In how many ways can a committee of 5 people be selected from 7 boys and 5 girls if at least 3 girls must be on the committee?

means 3 girls or 4 girls or 5 girls

case 1: 3 girls, 2 boys

$_5C_3 \cdot {}_7C_2 = 210$

case 2: 4 girls, 1 boy

$_5C_4 \cdot {}_7C_1 = 35$

case 3: 5 girls, 0 boys

$_5C_5 \cdot {}_7C_0 = 1$

Total = 210 + 35 + 1

= 246 ways

Ex. 9) On a geography exam Rihanna must answer 2 of the 4 questions in part A and at least 4 of 5 questions in part B. How many ways can she answer the questions?

case 1: 4 questions in pt B, 2 questions in part A

$_5C_4 \cdot {}_4C_2 = 30$

case 2: 5 questions in pt B, 2 questions in part A.

$_5C_5 \cdot {}_4C_2 = 6$

* Always add cases!

Total: 30 + 6
= 36 ways

Worksheet
#1, 2, 4, 5, 6, 8, 11, 12, 16

Assignment: Pg. 727; #4a,c, 5a,c, 7a,b, 8, 9, 12, 13, 15a,c, MC #1-3