Assignment: Logarithmic Graphs

1. Explain how the graphs of the functions below can be obtained from the graph of $y = \log_3 x$ (state the transformations)

   a) $y = \log_3(x - 1) + 3$

   b) $y = -2\log_3 x - 5$

   c) $y = \log_3(-2x + 4)$

2. Match each exponential function with the correct graph.

   a) $y = \log_3(x - 1)$

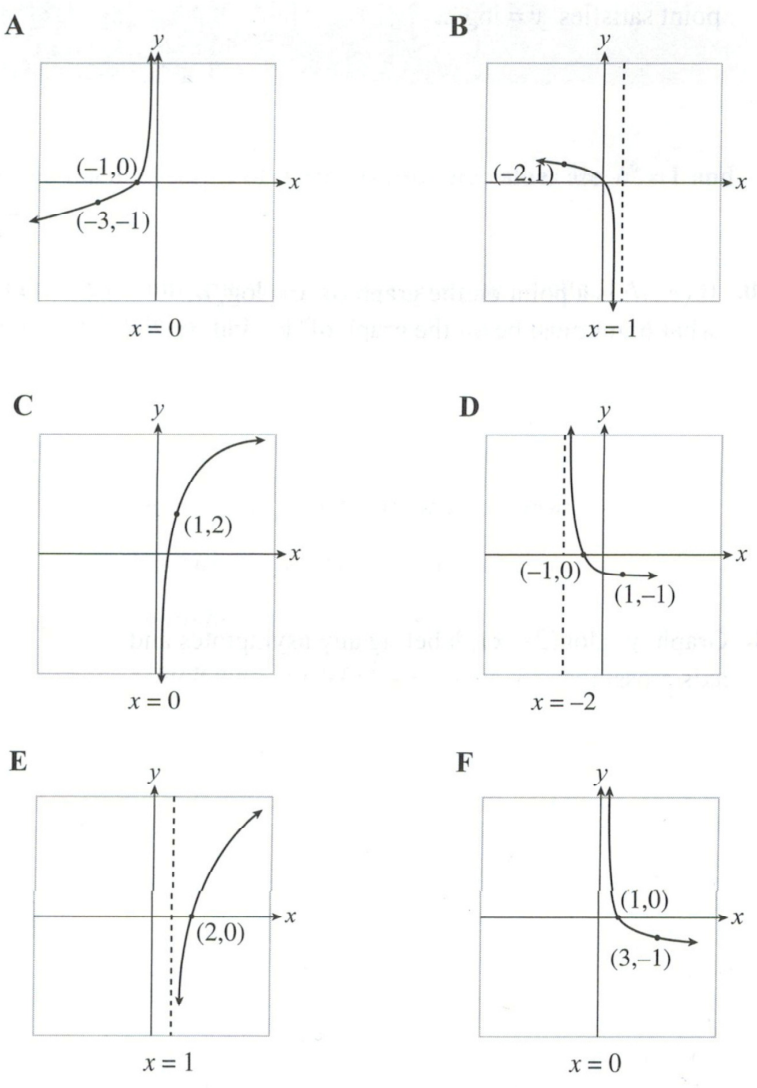
   b) $y = \log_3(1 - x)$

   c) $y = \log_3 x + 2$

   d) $y = -\log_3 x$

   e) $y = -\log_3(-x)$

   f) $y = -\log_3(x + 2)$



3. Sketch each of the following and state the domain, range, intercepts, and asymptotes.

   a) $y = \log_2(x - 3) + 1$

   b) $y = -\log_3 x - 1$

   c) $y = 3\log x - 4$

   d) $y = 4\log_{\frac{1}{3}}(x + 2)$

   e) $y = -\ln(x + 1)$

   f) $y = \ln(2x + 4) + 2$

Answers

1. Explain how the graphs of the functions below can be obtained from the graph of $y = \log_3 x$ (state the transformations)

a) $y = \log_3(x-1) + 3$ • Horizontal translation 1 unit right
• Vertical translation 3 units up

b) $y = -2\log_3 x - 5$
• reflect over x-axis, vert. stretch by 2, vert translation 5 down

c) $y = \log_3(-2x+4)$ $y = \log_3(-2(x-2))$
• reflect over y-axis, hor. comp by $\frac{1}{2}$, hor translation 2 right

2. Match each exponential function with the correct graph.

a) $y = \log_3(x-1)$

E (1,0) $\xrightarrow{+1}$ (2,0) V.A $x=1$
(3,1) → (4,1)

b) $y = \log_3(1-x)$ $y = \log_3(-(x-1))$

B (1,0) $\xrightarrow{+1}$ (-1,0) → (0,0)
(3,1) → (-3,1) → (-2,1)

c) $y = \log_3 x + 2$

C (1,0) $\xrightarrow{+2}$ (1,2)
(3,1) → (3,3)

d) $y = -\log_3 x$

F (1,0) → (1,0)
(3,1) → (3,-1)

e) $y = -\log_3(-x)$

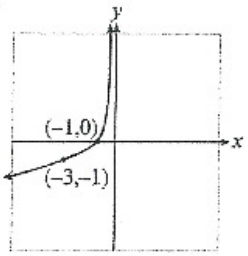
A (1,0) → (-1,0)
(3,1) → (-3,-1)

f) $y = -\log_3(x+2)$

D (1,0) → (-1,0) → (-1,0)
(3,1) → (3,-1) → (1,-1)

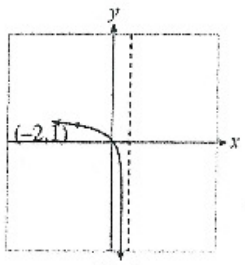
V.A $x = -2$

A



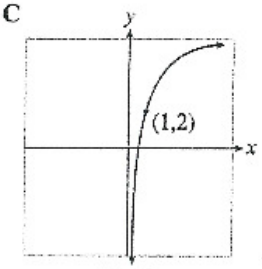
$x = 0$

B



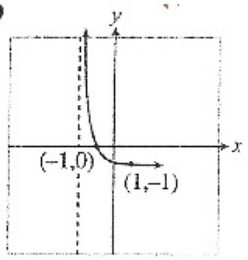
$x = 1$

C



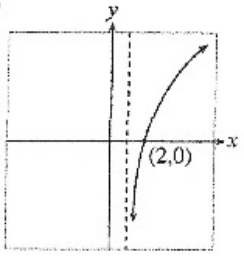
$x = 0$

D



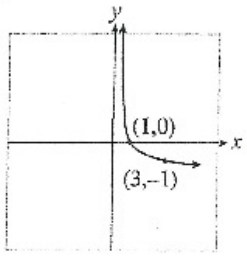
$x = -2$

E

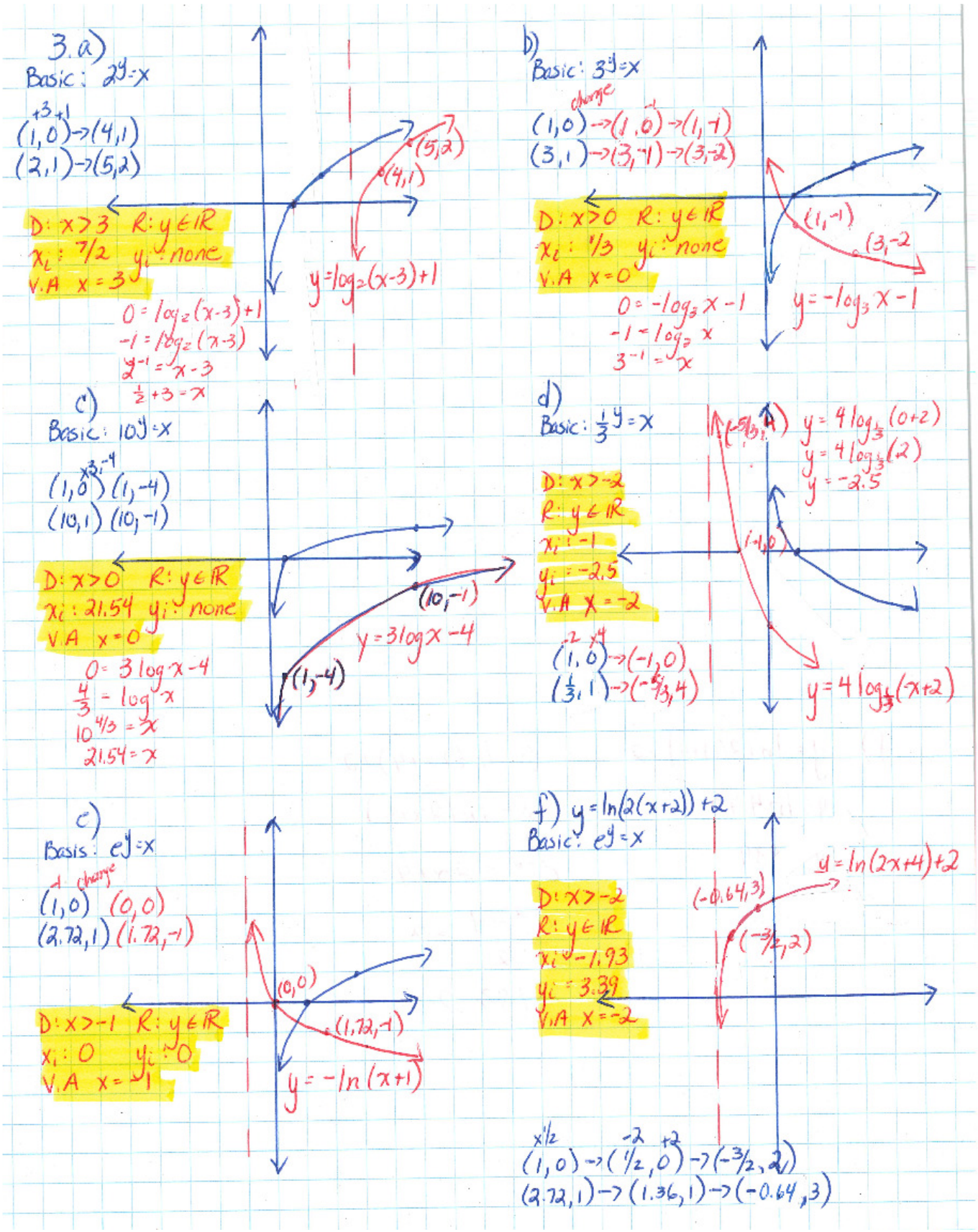


$x = 1$

F



$x = 0$


3. a)
Basic: $2^y = x$
$(1,0) \to (4,1)$
$(2,1) \to (5,2)$
D: $x > 3$ R: $y \in \mathbb{R}$
x_i: 7/2 y_i: none
V.A $x = 3$
$y = \log_2(x-3)+1$
$0 = \log_2(x-3)+1$
$-1 = \log_2(x-3)$
$2^{-1} = x-3$
$\frac{1}{2}+3 = x$
b)
Basic: $3^y = x$
$(1,0) \to (1,0) \to (1,-1)$
$(3,1) \to (3,-1) \to (3,-2)$
D: $x > 0$ R: $y \in \mathbb{R}$
x_i: 1/3 y_i: none
V.A $x = 0$
$y = -\log_3 x - 1$
$0 = -\log_3 x - 1$
$-1 = \log_3 x$
$3^{-1} = x$
c)
Basic: $10^y = x$
$(1,0) \to (1,-4)$
$(10,1) \to (10,-1)$
D: $x > 0$ R: $y \in \mathbb{R}$
x_i: 21.54 y_i: none
V.A $x = 0$
$y = 3\log x - 4$
$0 = 3\log x - 4$
$\frac{4}{3} = \log x$
$10^{4/3} = x$
$21.54 = x$
d)
Basic: $\frac{1}{3}^y = x$
D: $x > -2$
R: $y \in \mathbb{R}$
x_i: -1
y_i: -2.5
V.A $x = -2$
$(1,0) \to (-1,0)$
$(\frac{1}{3},1) \to (-\frac{5}{3},4)$
$y = 4\log_{\frac{1}{3}}(x+2)$
$y = 4\log_{\frac{1}{3}}(0+2)$
$y = 4\log_{\frac{1}{3}}(2)$
$y = -2.5$
e)
Basis: $e^y = x$
$(1,0)$ $(0,0)$
$(2.72,1)$ $(1.72,-1)$
D: $x > -1$ R: $y \in \mathbb{R}$
x_i: 0 y_i: 0
V.A $x = -1$
$y = -\ln(x+1)$
f) $y = \ln(2(x+2))+2$
Basic: $e^y = x$
D: $x > -2$
R: $y \in \mathbb{R}$
x_i: -1.93
y_i: 3.39
V.A $x = -2$
$y = \ln(2x+4)+2$
$(1,0) \to (\frac{1}{2},0) \to (-\frac{3}{2},2)$
$(2.72,1) \to (1.36,1) \to (-0.64,3)$