Assignment: Exponential Functions

1. Explain how the graphs of the functions below can be obtained from the graph of $y = 3^x$ (state the transformations).

    a) $y = 3^{x+2} - 3$

    b) $y = 2 \cdot 3^{2(x-4)} + 4$

    c) $y = -\frac{1}{5} \cdot 3^{-x}$

2. Match each exponential function with the correct graph.
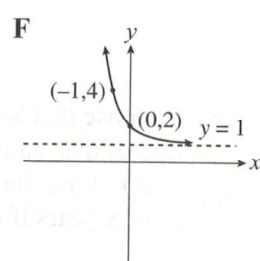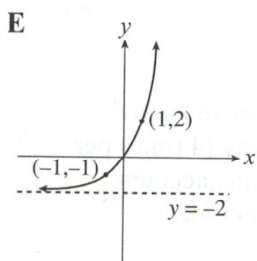
    a) $y = 4^{-x}$

    b) $y = 3^{-x+1}$

    c) $y = 3^{-x} + 1$

    d) $y = -4^{-x}$

    e) $y = 2^{x+1} - 2$

    f) $y = -2^{x+2} + 1$

MAPC40S   Exponential and Logarithmic Functions
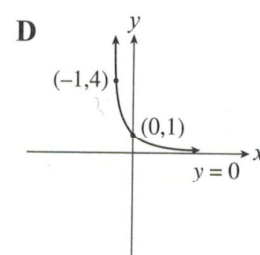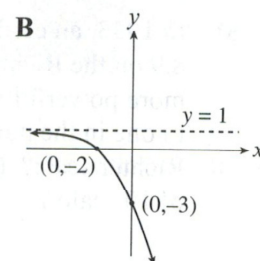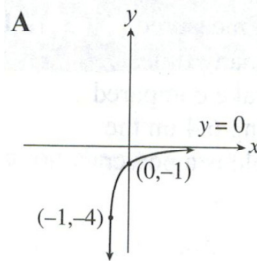
3. Sketch each of the following and state the domain, range, intercepts and asymptotes.

a) $y = 2^x + 2$

b) $y = -3^{x-1}$

c) $y = \left(\frac{1}{2}\right)^{x-2} - 1$

d) $y = 2 \cdot \left(\frac{1}{3}\right)^{-x}$

e) $y = 2^{\frac{1}{2}(x+1)} - 3$

f) $y = -e^{x+4} + 3$

Answers

1. Explain how the graphs of the functions below can be obtained from the graph of $y = 3^x$ (state the transformations).

a) $y = 3^{x+2} - 3$
   - Horizontal translation 2 units left
   - Vertical translation 3 units down

b) $y = 2 \cdot 3^{2(x-4)} + 4$
   - Vert stretch by 2
   - Hor. comp by ½
   - Horizontal translation 4 right
   - Vertical translation 4 up

c) $y = -\frac{1}{5} \cdot 3^{-x}$
   - Reflect over x-axis
   - vert comp by $\frac{1}{5}$
   - reflect over y-axis

2. Match each exponential function with the correct graph.

a) $y = 4^{-x}$

D switch signs $(0,1) \rightarrow (0,1)$ Inc
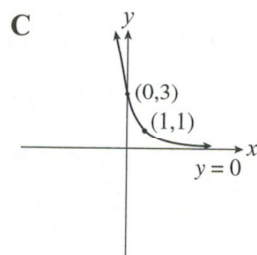 $(1,4) \rightarrow (-1,4)$

b) $y = 3^{-x+1}$ $y = 3^{-(x-1)}$

C switch signs $(0,1) \rightarrow (0,1) \rightarrow (1,1)$
          $(1,3) \rightarrow (-1,3) \rightarrow (0,3)$

c) $y = 3^{-x} + 1$

F switch signs $(0,1) \rightarrow (0,2)$
          $(1,3) \rightarrow (-1,4)$ H.A $y=1$

d) $y = -4^{-x}$

A $(0,1) \rightarrow (0,-1)$
   $(1,4) \rightarrow (-1,-4)$

e) $y = 2^{x+1} - 2$

E $(0,1) \rightarrow (-1,-1)$ H.A $y=-2$
   $(1,2) \rightarrow (0,0)$

f) $y = -2^{x+2} + 1$

B -2 if change sign +1
   $(0,1) \rightarrow (-2,-1) \rightarrow (-2,0)$
   $(1,2) \rightarrow (-1,-2) \rightarrow (-1,-1)$

   H.A $y = 1$

3.a)

Basic: $y=2^x$
$(0,1)$ $(1,2)$ $^{+3}_{+3}$
$(0,3)$ $(1,4)$

$y=2^x+2$
$(1,4)$
$(0,3)$

D: $x \in \mathbb{R}$ H.A $y=2$
R: $y>2$
x-int none
y-int 3

b) Basic: $y=3^x$
$H \to$ switch signs
$(0,1) \to (1,-1)$
$(1,3) \to (2,-3)$

$y=-3^{0-1}$
$y=-3^{-1}$
$y=-\frac{1}{3}$

$2(1,-1)$
$(2,-3)$
$y=-3^{x-1}$

D: $x \in \mathbb{R}$ H.A $y=0$
R: $y<0$
x_i: none
$y_i = -\frac{1}{3}$

c)

Basic: $y=\frac{1}{2}^x$
$(0,1) \to (2,0)$
$(1,\frac{1}{2}) \to (3,-\frac{1}{2})$

$y=\left(\frac{1}{2}\right)^{x-2}-1$
$(2,0)$
$(3,-\frac{1}{2})$

y-int $x=0$
$y=\left(\frac{1}{2}\right)^{0-2}-1$
$y=\left(\frac{1}{2}\right)^{-2}-1$
$y=4-1$
$y=3$

D: $x \in \mathbb{R}$ R: $y>-1$
x_i: 2
y_i: 3
H.A $y=-1$

d)

Basic: $y=\frac{1}{3}^x$
switch sign
$(0,1) \to (0,2)$
$(1,\frac{1}{3}) \to (-1,\frac{2}{3})$
$(-1,\frac{2}{3})$

$y=2\left(\frac{1}{3}\right)^{-x}$
$(0,2)$

D: $x \in \mathbb{R}$ R: $y>0$
x_i: none y_i: 2
H.A $y=0$

$y=2 \cdot \left(\frac{1}{3}\right)^{-(0)}$
$y=2 \cdot 1$
$y=2$

e)

Basic: $y=2^x$
$(0,1) \to (-1,-2)$
$(1,2) \to (1,-1)$

$y=2^{\frac{1}{2}(0+1)}-3$
$y=2^{\frac{1}{2}}-3$
$y=-1.6$

$(-1,-2)$ $(1,-1)$

$y=2^{\frac{1}{2}(x+1)}-3$

$0=2^{\frac{1}{2}(x+1)}-3$
$3=2^{\frac{1}{2}x+\frac{1}{2}}$
$\log 3 = \left(\frac{1}{2}x+\frac{1}{2}\right)\log 2$
$\log 3 = \frac{1}{2}x\log 2 + \frac{1}{2}\log 2$
$\log 3 - \frac{1}{2}\log 2 \div \frac{1}{2}\log 2 = x$
$2.17 = x$

D: $x \in \mathbb{R}$ R: $y>-3$
x_i: 2.2 y_i: -1.6
H.A $y=-3$

f)

Basic: $y=e^x$

$(-4,2)$
$(-3,0.72)$

$y=-e^{x+4}+3$

$(0,1) \to (0,-1) \xrightarrow{-4 \quad +3} (-4,2)$
$(1,2.72) \cdot (1,-2.72) \longrightarrow (-3,0.72)$

D: $x \in \mathbb{R}$
R: $y<3$
x_i: -2.9
y_i: -51.6
H.A $y=3$