Pre-Calculus 11 Sequences & Series

Lesson 1 Arithmetic Sequences

Consider the following sequence:

4, 7, 10, 13, 16, 19, … t_n

t_1 t_2 t_3 t_4

What is the pattern? add 3 to each term

This sequence is called an arithmetic sequence because each term is 3 larger than the previous term.

Definition:

An ***arithmetic sequence*** is a sequence in which each term after the first is obtained by adding or subtracting a fixed amount to the previous term.

The fixed amount that we are adding or subtracting each time is called the ***common difference*** and is denoted by the letter ***d***.

If the first term of this sequence is: t_1,
then the second term is: $t_1 + d$ $\quad 4 + 3(1) = 7$
the third term is: $t_1 + 2d$ $\quad 4 + 3(2) = 10$
the fourth term is: $t_1 + 3d$, and so on. $\quad 4 + 3(3) = 13$

$t_n = t_1 + (n-1)(d)$

$t_2 = 4 + (2-1)(3)$

$t_3 = 4 + (3-1)(3)$

$t_4 = 4 + (4-1)(3)$

General Form of an Arithmetic Sequence

The n^{th} term, t_n, of an arithmetic sequence with first term, t_1 and common difference, d is:

$$t_n = t_1 + d(n-1)$$

ie

$t_n = 4 + 3(n-1)$

Example 1 – Write the first 5 terms of:

a) an increasing arithmetic sequence 2, 6, 10, 14, 18 $\quad t_1 = 2 \quad d = 4$

b) a decreasing arithmetic sequence 12, 10, 8, 6, 4 $\quad t_1 = 12 \quad d = -2$

Examples: Calculate terms in a given arithmetic sequence

1. For this arithmetic sequence: $-3, 2, 7, 12, \ldots$ (+5, +5, +5; $\leftarrow d$; t_1)
   a) determine t_{20}. ($\leftarrow n$)
   b) Determine which term in the sequence has the value 212. ($\rightarrow$ solve for n; $\uparrow t_n$)

a) $t_n = t_1 + (n-1)d$

$t_{20} = -3 + (20-1)(5)$

$= 92$

b) $t_n = t_1 + (n-1)d$

$212 = -3 + (n-1)(5)$

$\frac{215}{5} = \frac{5(n-1)}{5}$

$43 = n-1$

$44 = n$

$\therefore t_{44} = 212$

or 212 is the 44^{th} term

2. For this arithmetic sequence: $3, 10, 17, 24, \ldots$
   a) determine t_{15}.
   b) determine which term in the sequence has the value 220.

a) $t_n = t_1 + (n-1)d$

$t_{15} = 3 + (15-1)(7)$

$= 101$

b) $t_n = t_1 + (n-1)d$

$220 = 3 + (n-1)(7)$

$217 = 7(n-1)$

$31 = n-1$

$32 = n$

$\therefore t_{32} = 220$

3. Two terms in an arithmetic sequence are $t_4 = -4$, and $t_7 = 23$. Determine t_1.

$-9, -9, -9$ (+d, +d, +d)

$\_, -22, -13, -4, \_, \_, 23$

$-4 = t_4$, $23 = t_7$

$t_7 = t_4 + 3d$

$23 = -4 + 3d$

$27 = 3d$

$9 = d$

$t_1 = -31$

or use the formula $t_n = t_1 + d(n-1)$

$23 = t_1 + 9(7-1)$

$-31 = t_1$

4. Two terms in an arithmetic sequence are $t_4 = -4$, and $t_7 = 23$. Determine t_{11}.

$t_n = t_1 + (n-1)d$

$t_{11} = -31 + (11-1)(9)$

$= 59$

Exercise 1 Arithmetic Sequences

1.) For this arithmetic sequence: $4, 7, 10, \ldots$ (follow example 1 or 2)
   a.) determine t_{15}. 46
   b.) determine which term in the sequence has the value 439. 146^{th}
2.) For this arithmetic sequence: $-3, 1, 5, \ldots$ (follow example 1 or 2)
   a.) determine t_{17}. 61
   b.) determine which term in the sequence has the value 97. 26^{th}
3.) Two terms in an arithmetic sequence are $t_{13} = -3$, and $t_{20} = -17$. Determine t_1. (follow example 3) 21
4.) Two terms in an arithmetic sequence are $t_6 = 10$, and $t_{18} = 46$. Determine t_{32}. (follow example 4) 88
5.) Write the first 5 terms of an arithmetic sequence with $a = 4$ and $d = 2$.
6.) Write the first 5 terms of an arithmetic sequence with $a = -5$ and d =-3/4.
7.) Determine x so that $x + 3$, $2x + 1$, and $5x + 2$ are consecutive terms of an arithmetic sequence. t_1 t_2 t_3

5) 4, 6, 8, 10, 12

6) $-5, -\frac{23}{4}, -\frac{13}{2}, -\frac{29}{4}, -8$

7) $-\frac{3}{2}$

4) $t_{18} = t_6 + 12d$
$46 = 10 + 12d$
$36 = 12d$
$3 = d$

$t_6 = t_1 + 5(3)$
$10 = t_1 + 15$
$-5 = t_1$

$t_{32} = -5 + (32-1)(3)$
$= 88$

7) $t_3 - t_2 = t_2 - t_1$
$5x+2-(2x+1) = 2x+1-(x+3)$
$5x+2-2x-1 = 2x+1-x-3$
$3x+1 = x-2$
$2x = -3$
$x = -\frac{3}{2}$

Extra practice: Pg. 8; #5, 6, 7, 10, 11a, b, 12