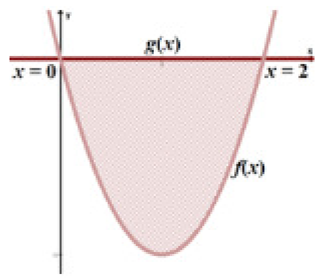
Integration Area Problem and Solution

Set up the **definite integral** that gives the following area (don't solve):

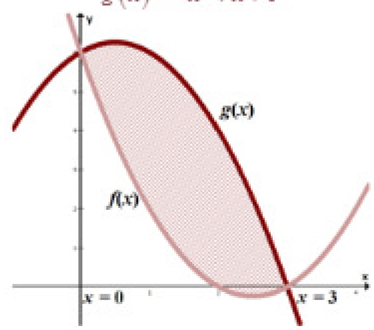
$$f(x) = x^2 - 2x$$
$$g(x) = 0$$



Solution: $\int_0^2 \left[0 - (x^2 - 2x)\right] dx = -\int_0^2 (x^2 - 2x)\, dx$

Set up and solve the **definite integral** that gives the following area:

$$f(x) = x^2 - 5x + 6$$
$$g(x) = -x^2 + x + 6$$



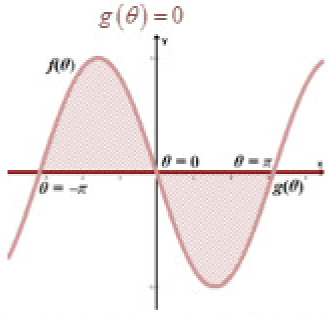
Solution:

$$\int_0^3 \left[(-x^2 + x + 6) - (x^2 - 5x + 6)\right] dx = \int_0^3 (-2x^2 + 6x)\, dx$$

$$= \left[-\frac{2}{3}x^3 + 3x^2\right]_0^3 = \left(-\frac{2}{3}(3)^3 + 3(3)^2\right) - \left(-\frac{2}{3}(0)^3 + 3(0)^2\right) = 9$$

Set up and solve the **definite integral** that gives the following area (don't solve):

$$f(\theta) = -\sin\theta$$
$$g(\theta) = 0$$



Solution: We need to divide graph into two separate integrals, since from $-\pi$ to 0, $f(\theta) \geq g(\theta)$, and from 0 to π, $g(\theta) \geq f(\theta)$:

$$\int_{-\pi}^0 (-\sin\theta - 0)\, d\theta + \int_0^\pi \left[0 - (-\sin\theta)\right] d\theta = \int_{-\pi}^0 (-\sin\theta)\, d\theta$$

$$+ \int_0^\pi (\sin\theta)\, d\theta = \left[\cos x\right]_{-\pi}^0 + \left[-\cos x\right]_0^\pi = \cos(0)$$

$$-\cos(-\pi) + \left[-\cos(\pi) + \cos(0)\right] = 1 - (-1) + (1 + 1) = 4$$

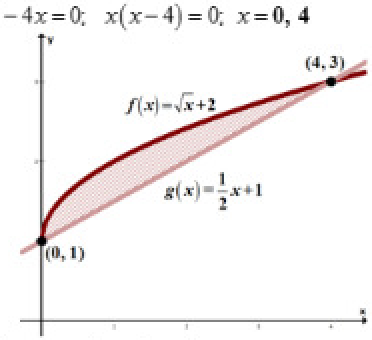
Sketch the region bounded by the graphs, and **find the area**:

$$f(x) = \sqrt{x} + 1, \;\; g(x) = \frac{1}{2}x + 1$$

Solution: Let's draw the curves and set them equal to each other to see where the limits of integration will be:

$$\sqrt{x} + 1 = \frac{1}{2}x + 1; \;\; \sqrt{x} = \frac{1}{2}x; \;\; x = \frac{x^2}{4}; \;\; 4x = x^2$$

$$x^2 - 4x = 0; \;\; x(x - 4) = 0; \;\; x = \mathbf{0,\ 4}$$



Now let's integrate from 0 to 4:

$$\int_0^4 \left[(\sqrt{x} + 1) - \left(\frac{1}{2}x + 1\right)\right] dx = \int_0^4 \left(x^{\frac{1}{2}} - \frac{x}{2}\right) dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{4}x^2\right]_0^4 = \left[\frac{2}{3}(4)^{\frac{3}{2}} - \frac{1}{4}(4)^2\right] - 0 = \frac{4}{3}$$

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