Lesson 4 Graphing Reciprocals of Quadratic Functions

When we graph the reciprocal of a quadratic function, the quadratic function may have 0, 1, or 2 vertical asymptotes.

There are 3 basic shapes

Shape 1 – Funnel or Inverted Funnel

- This shape has one vertical asymptote

Shape 2 – H-Shape

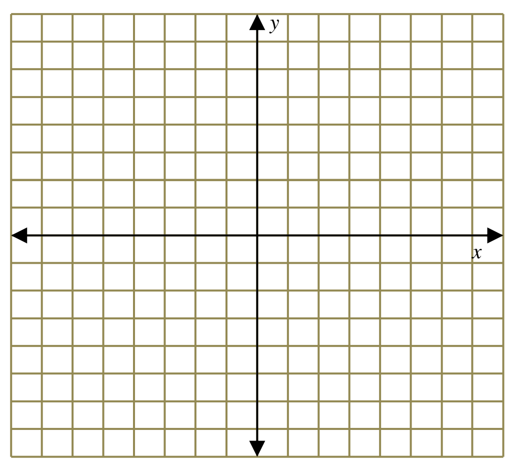
- This shape has two vertical asymptotes

Shape 3 – The Speed Bump or Pot Hole

- This shape has no vertical asymptote

Examples
Funnel or Inverted Funnel (One Vertical Asymptote)

1. Sketch $y = \frac{1}{(x-1)^2}$.



Step 1: Sketch the graph $y = (x-1)^2$

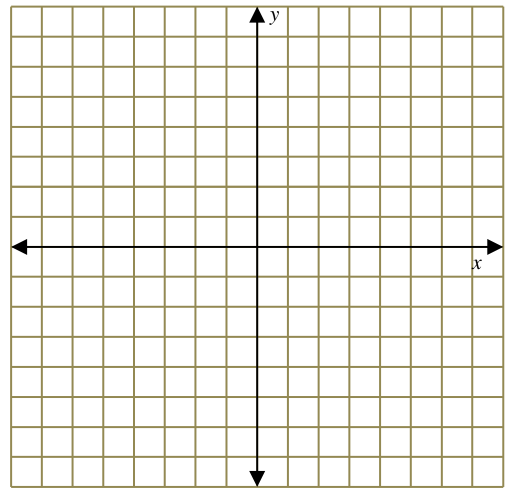
Step 2: Sketch vertical asymptotes at the *x*-intercepts. ie. at the restrictions on the denominator.

Note: the horizontal asymptote is the *x*-axis since reciprocals of positive values will be positive and reciprocals of negative values will be negative.

Step 3: Plot the invariant points. Where $y = \pm 1$

Step 4: Sketch the graph, approaching the asymptotes

2. Sketch $y = \frac{1}{-(x+2)^2}$.

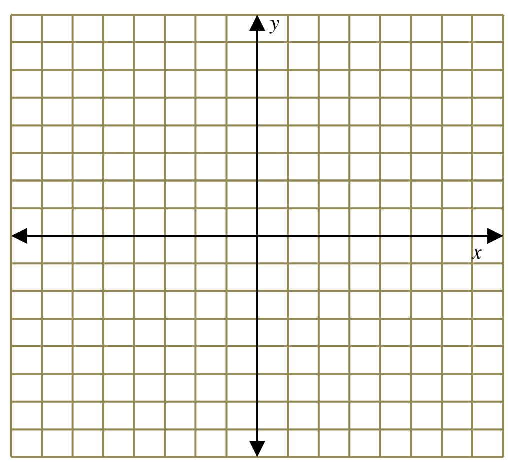


H-Shape (Two Vertical Asymptotes)

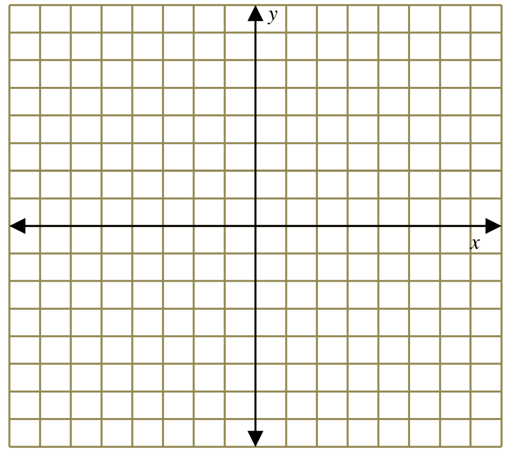
Steps:

1. Sketch the quadratic function
2. Sketch vertical asymptotes through the x-intercepts
3. Plot the invariant points
4. Plot the reciprocal of the main points.
5. Sketch the graph.
6. Remember to erase the original graph or clearly label

3. Sketch $y = \frac{1}{x^2-4}$

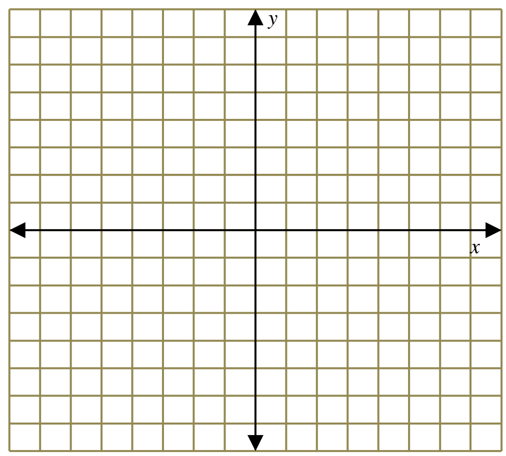


4. Sketch $y = \frac{1}{(x-1)(x+1)}$



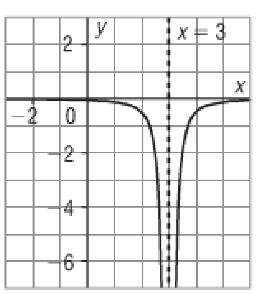
Speed Bump or Pot Hole (No Vertical Asymptote)

5. Sketch $y = \frac{1}{x^2+3}$



Given the graph of each reciprocal function $y = \frac{1}{f(x)}$, sketch $y = f(x)$.

a)



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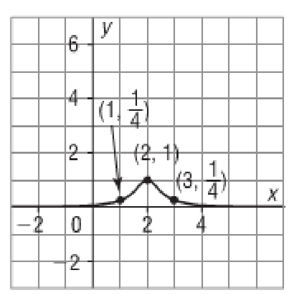


$y = \frac{1}{f(x)}$

Assignment 4 Reciprocals of Quadratic Functions__________

1.) Sketch the following reciprocal graphs.

a.) $y = \frac{1}{(x-3)^2}$

b.) $y = \frac{1}{-(x+4)^2}$

c.) $y = \frac{1}{3(x-1)^2}$

d.) $y = \frac{1}{x^2-3}$

e.) $y = \frac{1}{-x^2+1}$

f.) $y = \frac{1}{-x^2+4}$

g.) $y = \frac{1}{x^2-2x-8}$

h.) $y = \frac{1}{x^2+2}$

Assignment 5 Reciprocals of Quadratic Functions

1. Given the function $y = f(x)$, write the corresponding reciprocal function.

   a) $y = x^2 - 9$

   b) $y = x^2 - 7x + 10$

2. For each function;

   - State the zeros
   - Write the reciprocal function
   - State the non-permissible values of the corresponding rational expression
   - State the equation(s) of the vertical asymptote(s)

   a) $f(x) = x^2 - 16$

   b) $g(x) = x^2 + x - 12$

3. State the equation(s) of the vertical asymptote(s) for each function.

   a) $f(x) = \frac{1}{(x-2)(x+4)}$

   b) $f(x) = \frac{1}{x^2-9x+20}$

   c) $f(x) = \frac{1}{-x^2-5}$

4. Determine the domain of the following functions:

   a) $f(x) = \frac{1}{(x+1)(x-3)}$

   b) $f(x) = \frac{1}{x^2+8}$

   c) $f(x) = \frac{1}{-(x-5)^2}$

5. What are the x-intercept(s) and the y-intercept of each function?

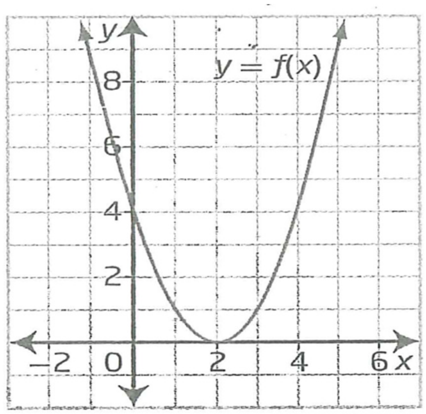
   a) $f(x) = \frac{1}{x^2-9}$

   b) $f(x) = \frac{1}{x^2+7x+12}$

   c) $f(x) = \frac{1}{(x-2)^2}$

6. Given the graphs of $y = f(x)$, sketch the graph of the reciprocal function $y = \frac{1}{f(x)}$. Describe your method.

   a)



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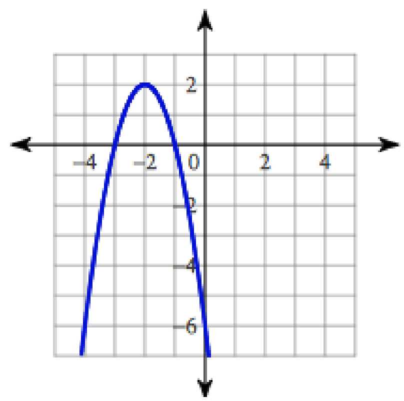

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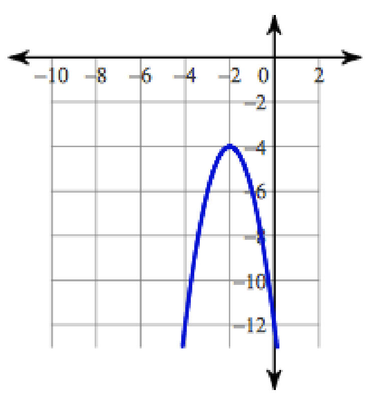
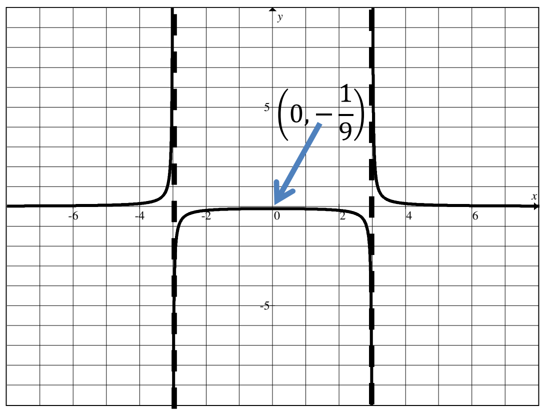


7. Each of the following is the graph of a reciprocal function, $y = \frac{1}{f(x)}$.

- Sketch the graph of the original function, $y = f(x)$

a)



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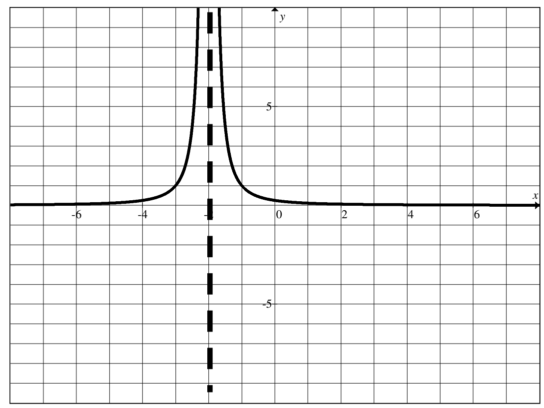
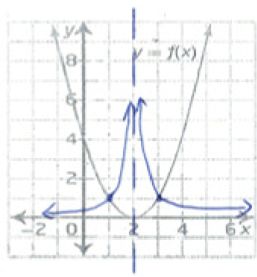


Answer Key:

1. a) $y = \frac{1}{x^2-9}$ b) $y = \frac{1}{x^2-7x+10}$
2. a) $x = -4, x = 4;\ y = \frac{1}{x^2-16};\ x \neq -4, 4,;\ x = -4, x = 4$
   b) $x = 3, x = -4;\ y = \frac{1}{x^2+x-12};\ x \neq 3, -4;\ x = 3, x = -4$
3. a) $x = 2, -4$ b) $x = 4, 5$ c) no vertical asymptotes
4. a) no x-intercepts, y-int: $-\frac{1}{9}$ b) no x-intercepts, y-int: $\frac{1}{12}$
   c) no x-intercepts, y-int: $\frac{1}{4}$
5. a) $D: x \neq -1, 3\ or\ (-\infty, -1) \cup (1,3) \cup (3, \infty)$
   b) $D: x \in \mathbb{R}, or\ \ (-\infty, \infty)$
   c) $D: x \neq 5, or\ (-\infty, 5) \cup (5, \infty)$

6.)

a)



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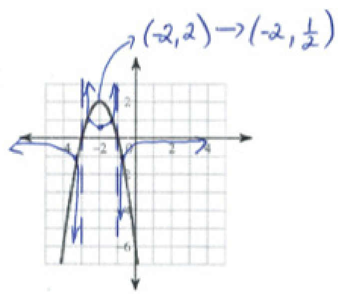

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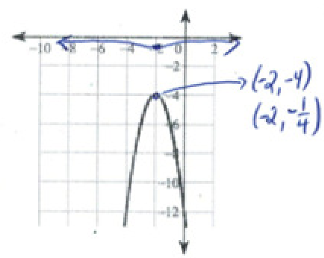
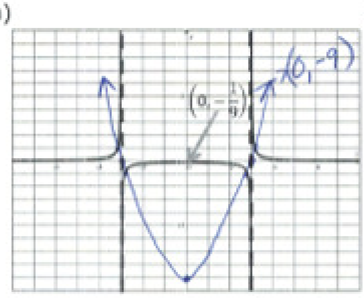


7.)

a)



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