Lesson 3 Reciprocals of Linear Functions

Reciprocal Numbers: A reciprocal of any number, a is represented by

$$y = \frac{1}{a}, a \neq 0$$

Reciprocal Functions: A reciprocal of any function, $f(x)$, is represented by

$$y = \frac{1}{f(x)}, f(x) \neq 0$$

Asymptotes are lines the graph approaches but never crosses/touches as the values of either x or y approach infinity or negative infinity.
Asymptotes are not part of the graph and therefore are represented by broken lines.

Example 1

Determine the equation of the vertical asymptote on the graph of $y = \frac{1}{-5x+4}$.

$-5x + 4 = 0$ ← set denom equal to 0

$4 = 5x$

$\frac{4}{5} = x$

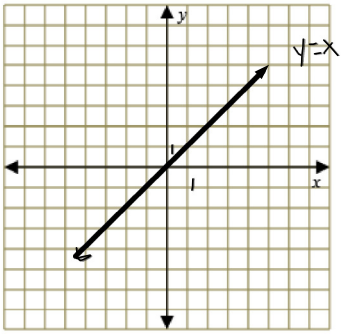
Example 2

Sketch $y = \frac{1}{x}$

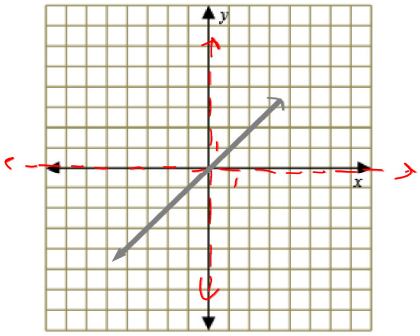
Step 1: Sketch $y = x$

y-int 0

m = 1



Step 2: ***Vertical asymptote*** – through x-int(s)
This is the restriction of the rational expression. The denominator equals 0.
∴ a ***Horizontal asymptote*** occurs at $y = 0$.

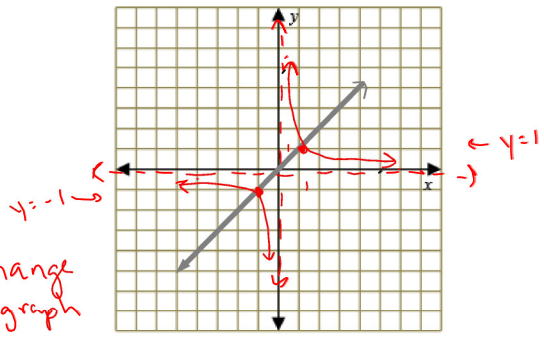


* The reciprocal of 0 is undefined.

Step 3: Place points wherever $y = \pm 1$ on the original function. These are the points that are same when you take the reciprocal. $1 = \frac{1}{1}$, and $-1 = \frac{1}{-1}$

These points are called ***invariant points***.

points that don't change from the original graph to another graph



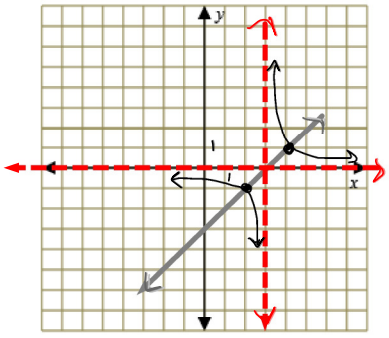
Step 4: Sketch the reciprocal graph. (shape is called a hyperbola)

Remember what happens in basic division of numbers. As the denominator becomes larger, the resulting number becomes smaller.
As the denominator becomes smaller, the resulting number becomes larger.
The graph approaches the asymptotes towards $\pm\infty$.

Example 3
Sketch the following rational functions.

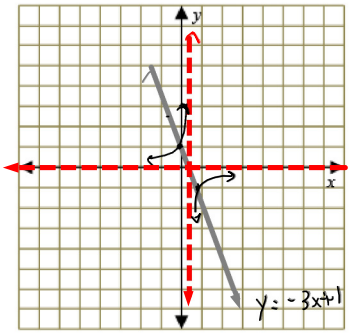
a) $y = \frac{1}{x-3}$

① Sketch $y = x - 3$
② Asymptotes H.A. $y = 0$
V.A. $x = 3$ ← through x-int
③ Invariant pts
$y = \pm 1$



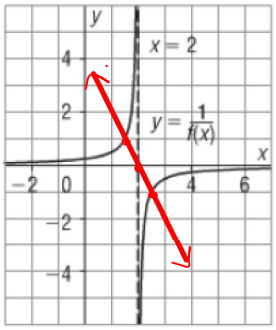
b) $y = \frac{1}{-3x+1}$

Sketch $y = -3x+1$



Example 4

Given the graph of $y = \frac{1}{f(x)}$, sketch the graph of $y = f(x)$.



x-intercept where V.A. lies

invariant pts $y = \pm 1$