Lesson 1 Absolute Value Functions

Absolute Value

The absolute value of a value, a, is the distance on the real number line between zero and a. Since the distance between 0 and a is the same as the distance between 0 and $-a$, we denote this as $|a|$.

$|2| = 2$ $|-2| = 2$

The absolute value of a is defined as

$$|a| = \begin{cases} a, \text{if } a \geq 0 \\ -a, \text{if } a < 0 \end{cases}$$

Absolute Value Function

The absolute value of a function $f(x)$ is defined as

$$|f(x)| = \begin{cases} f(x), \text{if } f(x) \geq 0 \\ -f(x), \text{if } f(x) < 0 \end{cases}$$

and therefore the graph of $y = |f(x)|$ is $\begin{cases} y = f(x), \text{if } x \geq 0 \\ y = -f(x), \text{if } x < 0 \end{cases}$

making it a piecewise function.

A piecewise function is a function which is defined by multiple sub-functions, each function applying to a sub-domain of the function's domain.

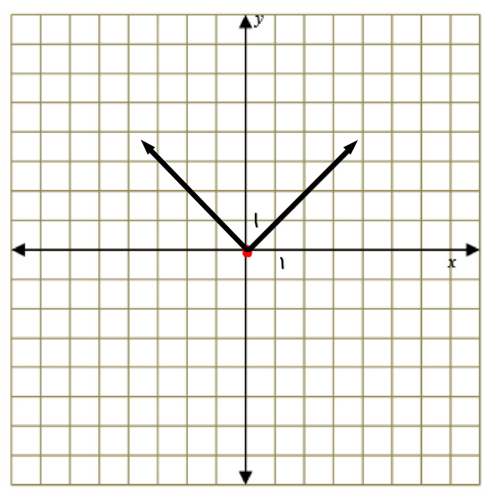
Graph $y = |x|$

The graph of $y = |x|$ is the graph of $y = x$ on one sub-interval of the domain and $y = -x$ on a different sub-interval, connected at the critical point (0, 0).

We could write this in piecewise notation as

$$y = \begin{Bmatrix} x, \text{if } x \geq 0 \\ -x, \text{if } x < 0 \end{Bmatrix}$$

$y = x$
$y = -x$

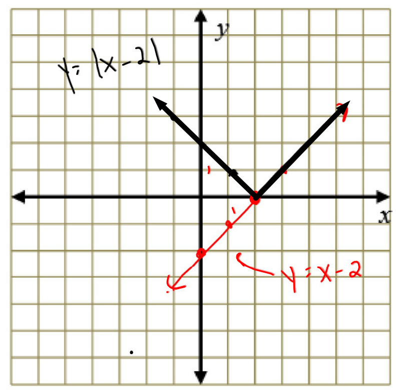


To graph the absolute value of a linear function, graph the line (using slope-intercept method) and then reflect all the negative values of y since absolute value makes all y-values positive.

Examples

Sketch the graphs of the following absolute value functions. Identify the intercepts, domain, and range of each function.

1. $y = |x - 2|$



Written as a piecewise function, this is

$$y = \begin{cases} x - 2, & if\ x \geq 2 \\ -x + 2, & if\ x < 2 \end{cases}$$

x-intercept or critical value

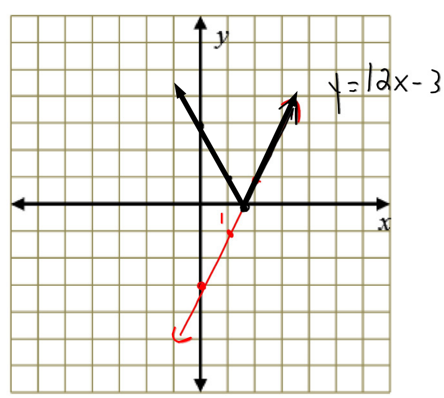
Sketch $y = x - 2$

occurs when $y = 0$

$\therefore\ 0 = x - 2$

$x = 2$

2. $y = |2x - 3|$



Steps $y = mx + b$

① Plot y-int (b) -3

② Use slope (m) to get second pt $\frac{2}{1}$ up 2 right 1

Sketch $y = 2x - 3$

x-int

$0 = 2x - 3$

$3 = 2x$

$\frac{3}{2} = x$

keep all positive values of y, reflect all negative values of y

In piecewise notation

$$y = \begin{cases} 2x - 3 & x \geq \frac{3}{2} \\ -2x + 3 & x < \frac{3}{2} \end{cases}$$

$0 = \frac{-3}{2}x + 1$

$y = -\frac{3}{2}x + 1$

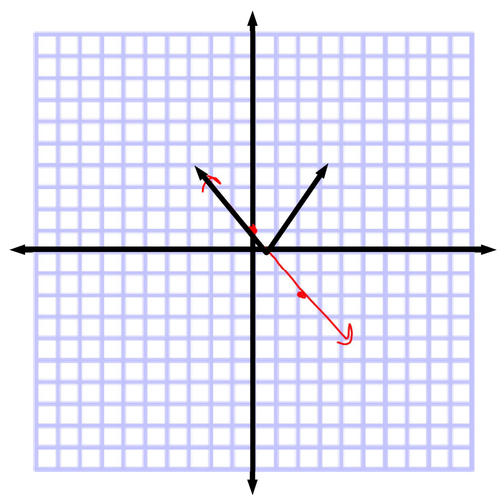
$-1 = \frac{-3}{2}x$

$y = \left| \frac{-3}{2}x + 1 \right|$

$-2 = -3x$

$\frac{2}{3} = x$

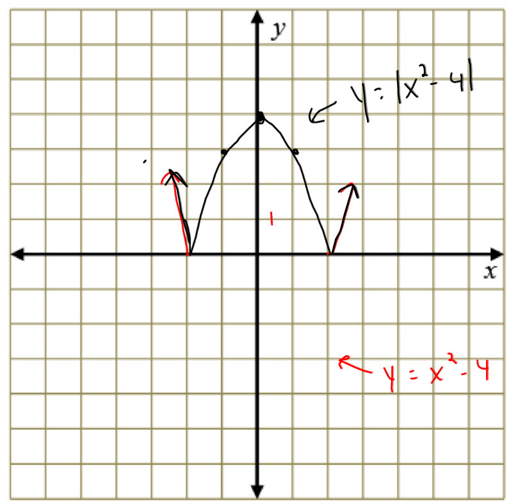
$$y = \begin{cases} -\frac{3}{2}x + 1 & x < \frac{2}{3} \\ \frac{3}{2}x - 1 & x \geq \frac{2}{3} \end{cases}$$



To graph the absolute value of a quadratic function, graph the parabola, using transformations and then reflect all the negative values of y since absolute value makes all y-values positive.

Examples
For each function, sketch its graph and determine its intercepts, domain and range.

1. $y = |x^2 - 4|$



Written as a piecewise function, this is

$$y = \begin{cases} x^2 - 4, & if\ x \le -2 \text{ or } x \ge 2 \\ -x^2 + 4, & if\ -2 < x < 2 \end{cases}$$

Sketch $y = x^2 - 4$

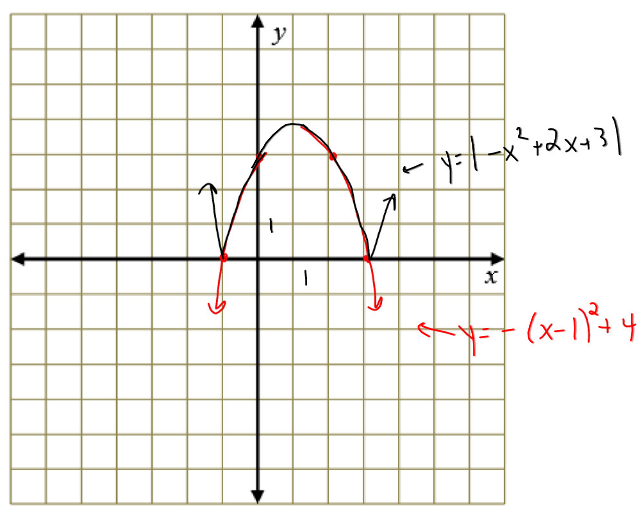
keep positive values of y,
reflect negative values of y

2. $y = |-x^2 + 2x + 3|$

$y = -x^2 + 2x + 3$

$y = -(x^2 - 2x + 1) + 3 + 1$

$y = -(x-1)^2 + 4$



As a piecewise function

$$y = \begin{cases} -x^2 + 2x + 3 & -1 \le x \le 3 \\ x^2 - 2x - 3 & x < -1 \text{ or } x > 3 \end{cases}$$

Using Transformations

If the entire function is not enclosed in absolute value bars, sketch the function in the absolute value bars and then apply transformations.

Examples

Sketch the given functions.

1. $y = -2|x - 1| + 3$

Start w/ $y = |x-1|$

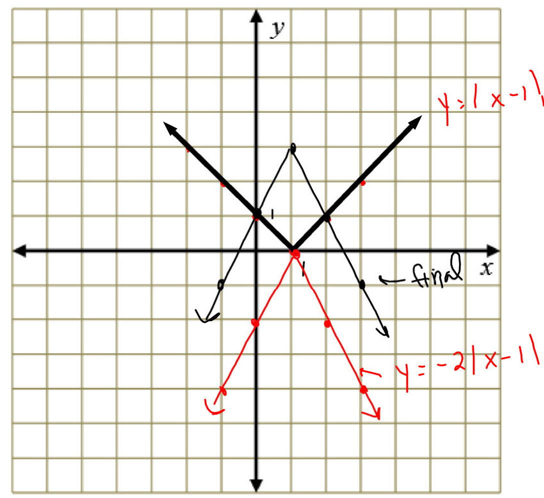
$y = -2|x-1|$

vertical reflection

vertical stretch by a factor 2

$y = -2|x-1| + 3$

vertical translation up 3 units

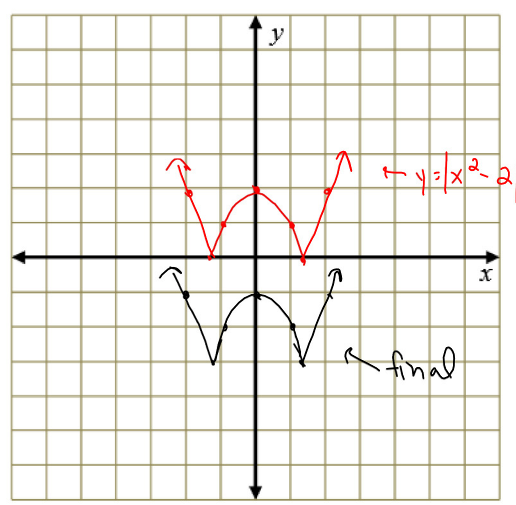


2. $y = |x^2 - 2| - 3$

$y = |x^2 - 2|$

$y = |x^2 - 2| - 3$

down 3



pg 182 # 1a, c, h
2l
3a, f

Sketch
$y = |-x+3|$
$y = |-x^2-1|$
$y = |(x+1)^2-4|$