

MA PC 405

Logarithmic Applications

23. Solve:

- a) An average new car depreciates 15% in value each year. How long does it take for a new \$40 000 car to depreciate down to \$10 000 in value?
- b) If you have \$10 000 and invest the full amount at 12% compounded quarterly, how long does it take you to become a millionaire?
- c) What interest rate compounded semi-annually is needed if money is to triple in 15 years?
- d) It is estimated that 20% of a certain radioactive substance decays in 30 hours. What is the half-life of this substance?
- e) In chemistry, the pH of a substance is defined by $pH = -\log[H^+]$ where $[H^+]$ is the concentration of hydrogen ions in the substance, measured in moles per litre.
 - (i) Find the pH of orange juice if its $[H^+]$ concentration is given by 6.83×10^{-5} moles per litre.
 - (ii) What is the hydrogen ion concentration $[H^+]$ in moles per litre of a tomato if its pH is 4.21?
- f) If Vancouver has a population of 400 000 and is growing at a rate of 2% annually, and Surrey has a population of 300 000 and is growing at a rate of 3% annually, in how many years will Surrey catch up to Vancouver in population?
- g) Find the time needed for money to triple at 8% compounded continuously.
- h) The amount of a chemical in grams that will dissolve in a solution is given by $C = 8e^{0.3t}$ where t is the temperature in Celsius of the solution. Find t when $C = 100$ grams.
- i) The population of Toronto is given by $P(t) = 4\,000\,000 e^{0.012t}$, where $t = 0$ corresponds to year 2000. What year will the population reach 6 400 000?
- j) A biologist studying a colony of bacteria determines that a certain culture grows exponentially such that the bacteria doubles every 4 days. If initially the biologist has 1 200 bacteria present, how many days does it take before 100 000 bacteria are present?
- k) The half life of radioactive carbon 14 is 5 570 years. Assume it decays according to the equation $y = y_0 e^{kt}$. If 500 milligrams of radioactive carbon 14 are present today, determine the amount present after 2 500 years.