Pre-Calculus 12
Application of Logs

I. Growth and Decay

Doubling Period: The time it takes for a population/substance to double in size.

Half-Life: The time it takes for a population/substance to reduce to half its size. (decompose)

π – used in formulas to determine area and circumference of a circle

e – an irrational number (like π) whose value is 2.718… It's a special value used to determine growth of populations; formula for natural growth.

The Law of Natural Growth

This formula is used for continuous growth and decay.

$A = Pe^{rt}$

Where:

A = Final amount

P = Original amount

r = Growth Rate

t = Time

Examples

1. A radioactive substance decays at a daily rate of 0.13. How long does it take for this substance to decompose to half its size?

decreasing so rate is negative → −0.13

choose numbers such as P = 200, A = 100 (half of original)

choose numbers where A is ½ of P

$A = Pe^{rt}$

$100 = 200e^{-0.13t}$ — divide

$0.5 = e^{-0.13t}$

$\ln 0.5 = \ln e^{-0.13t}$ — Apply ln to both sides

$\ln 0.5 = -0.13t \ln e$ — Power law

$\frac{\ln 0.5}{-0.13} = t$ — Evaluate

$5.332 \text{ days} = t$

2. There are 500 gophers in a field on May 31st. On June 20th there are 800.

start with 500 (P) — time 20 days — new population 800 (A) (part b)

a) Find the rate of growth.

$A = Pe^{rt}$

$800 = 500e^{r(20)}$

$1.6 = e^{20r}$

$\ln 1.6 = \ln e^{20r}$

$\ln 1.6 = 20r \ln e$ ($\ln e = 1$)

$\frac{\ln 1.6}{20} = r$

store in calc → $0.0235\ldots = r$

b) How many gophers are there on June 28th?

$A = Pe^{rt}$

$A = 500e^{(0.0235\ldots(28))}$

$A = 965$ gophers

$500 \times 2^{nd}\ e^{\wedge}\ \ln\ (\text{ANS} \times 28)$

3. Lead-210 is a radioactive nuclide. If 8g of it decays to 6.75g in 5 years, then what is the half-life of lead-210?

Part 1 — start with P — end with A — t — Part 2

2 part question

Part 1 Determine r

$P = 8$

$A = 6.75$

$t = 5$

$r = ?$

$A = Pe^{rt}$

$6.75 = 8e^{r(5)}$

$0.84375 = e^{5r}$

$\ln 0.84375 = 5r \ln e$ ($\ln e = 1$)

$\frac{\ln 0.84375}{5} = r$

$-0.03397\ldots = r$

↑ store in calc

Part 2 Determine half-life (t)

$P = 8$

$A = 4$

$r = -0.0339\ldots$

$t = ?$

$A = Pe^{rt}$

$4 = 8e^{-0.0339\ldots t}$

$0.5 = e^{-0.0339\ldots t}$

$\ln 0.5 = -0.0339\ldots t \ln e$ ($\ln e = 1$)

$\frac{\ln 0.5}{-0.0339\ldots} = t$

$20.399 = t$

$\therefore$ 20.399 years

alternate forms of eqn: $A = Y_0 e^{-kt}$ or $Y_n = Y_0 e^{-kt}$

II. Earthquakes

The most intense earthquake ever recorded was in Chile in May 1960, with a magnitude of 9.5.

$M = \log\left(\frac{I}{S}\right)$ $\quad M = \log\frac{A}{A_0}$

Where: M is the magnitude
I (A) is the intensity of the ground motion
S (A_0) is the intensity of a standard earthquake

↳ doesn't have a value ie stays as S

a) Calculate the intensity of the earthquake in Chile in terms of a standard earthquake.

$M = \log\left(\frac{I}{S}\right)$

$9.5 = \log\left(\frac{I}{S}\right)$ Solve for I

$10^{9.5} = \frac{I}{S}$ Put in exp form

$10^{9.5}S = I$ Isolate I

b) An earthquake that occurred in Haiti was 10^7 times as intense as a standard earthquake. How many times as intense as the Haiti earthquake was the Chile earthquake? Give answer to the nearest whole number.

Haiti

$I = 10^7 S$

$\frac{\text{Chile}}{\text{Haiti}} = \frac{10^{9.5}S}{10^7 S}$ } set up ratio to compare

$= 10^{2.5}$

$= 316$ times as intense

worksheet d, e, h, j, k