Antiderivatives (cont'd)

ex.1 Find a curve that has, at each point (x, y), the slope $2x$.

$$\int 2x\,dx = x^2 + c$$

$\therefore y = x^2$ is a curve which has a slope $2x$ at each pt.

Antidifferentiation Using Substitution

Def'n Indefinite Integral
The family of all antiderivatives of a fcn. $f(x)$ is the indefinite integral of f w.r.t. x and is denoted by $\int f(x)\,dx$

Note $\int u^{-1}\,du = \int \frac{1}{u}\,du = \ln|u| + c$

$\frac{dy}{du} = u^{-1}$
$dy = u^{-1}\,du$
$\int u^{-1}\,du$

The Substitution Rule

If $u = g(x)$ is a differentiable fcn whose range is on interval I and f is continuous on I, then

$$\int \underbrace{f(g(x))\,g'(x)}_{\text{fcn and its derivative}}\,dx = \int f(u)\,du$$

ex.1 Evaluate

a) $\int \cos(\overset{u}{5x+3})(\overset{du}{5\,dx})$

$f(g(x))$ $g'(x)$

use substitution

$u = 5x+3$
$du = 5\,dx$

$\int \cos u\,du$

integrate

$\sin u + c$

$\sin(5x+3) + c$ ← sub back in

b) $\int (7x+4)^9\,dx$

$u = 7x+4$
$du = 7\,dx$
$\frac{du}{7} = dx$ isolate dx

Bring constants to the front

$\int u^9\left(\frac{du}{7}\right)$

$\frac{1}{7}\int u^9\,du$

$\frac{1}{7}\left(\frac{u^{10}}{10}\right) + c$

$\frac{(7x+4)^{10}}{70} + c$

c) $\int x^3 \cos(x^4+2)\,dx$

$u = x^4 + 2$
$du = 4x^3\,dx$
$\frac{du}{4} = x^3\,dx$ divide by 4 to match $x^3\,dx$ in the question

$\int \cos u\,\frac{du}{4}$ same as $\frac{1}{4}du$

$\frac{1}{4}\int \cos u\,du$

$\frac{1}{4}\sin u + c$

$\frac{1}{4}\sin(x^4+2) + c$

d) $\int \sqrt{2x+1}\,dx$

$\int (2x+1)^{\frac{1}{2}}\,dx$

$u = 2x+1$
$du = 2\,dx$
$\frac{du}{2} = dx$

$\frac{1}{2}\cdot\int u^{\frac{1}{2}}\,du$

integrate

$\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$

$u^{\frac{3}{2}} \div \frac{3}{2}$

$u^{\frac{3}{2}} \cdot \frac{2}{3}$

$\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}u^{\frac{3}{2}}\right) + c$

$\frac{u^{\frac{3}{2}}}{3} + c$

$\frac{(2x+1)^{\frac{3}{2}}}{3} + c$

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