Pre-Calculus 12 Analyzing Rational Functions

A rational function has the form $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$.

Characteristics of Rational Functions

1. Non-permissible values of *x*

- **Vertical Asymptote**: a vertical line that the graph approaches but never touches

$y = \frac{1}{(x-3)(x+1)}$

- Set the denominator equal to 0 and solve for *x*

- **Hole**: a point of discontinuity

$y = \frac{x^2-25}{x+5}$

- Factor (if possible)
- Divide out common factors, set common factor equal to 0 and solve for *x*-value of hole
- Solve remaining function for corresponding *y*-value

Ex. 1) Determine any non-permissible values

a) $y = \frac{x^2}{x^2-9}$

b) $y = \frac{x^2-x-2}{x+1}$

Horizontal Asymptotes (H.A.)

For $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ have no common factors, the following happens:

- If the degree of $p(x) <$ degree of $q(x)$ the H.A. is the line $y = 0$

$y = \frac{x+2}{x^2+6x-7}$

- If the degree of $p(x) =$ to the degree of $q(x)$ then the H.A. is the line $y = \frac{a}{b}$, where "a" is the leading coefficient of $p(x)$ and "b" is the leading coefficient of $q(x)$

$y = \frac{2x^2-1}{3x^2+5}$

- If the degree of $p(x) >$ the degree of $q(x)$ then there will be no H.A.

$\boldsymbol{y = \frac{x}{x^2+3}}$

Ex. 2) Determine the equations of any asymptotes and note the coordinates of any holes.

a) $y = \frac{2x}{x^2-4}$

b) $y = \frac{x^2+2x}{x^2-4}$

Ex. 3) Solve.

$$0 = \frac{3}{x-2} + 4$$

Assignment: Pg. 114; #4b, 5a, 6a,b, 8a (just solve like ex. 2), 10a,b (i, ii)