Pre-Calculus 12 Analyzing Rational Functions

A rational function has the form $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$.

Characteristics of Rational Functions

1. Non-permissible values of x

- **Vertical Asymptote**: a vertical line that the graph approaches but never touches

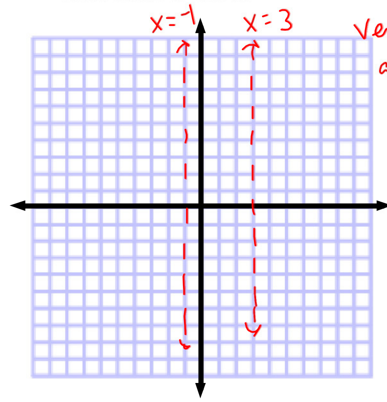
$y = \frac{1}{(x-3)(x+1)}$

$(x-3)(x+1) = 0$

$x - 3 = 0 \qquad x + 1 = 0$

$x = 3 \qquad x = -1$

- Set the denominator equal to 0 and solve for x



vertical asymptotes

- **Hole**: a point of discontinuity

$y = \frac{x^2-25}{x+5}$

$y = \frac{\cancel{(x+5)}(x-5)}{\cancel{x+5}}$

common factor

$x + 5 = 0$

$x = -5$

remaining function

$y = x - 5$

$y = -5 - 5$

$y = -10$

$\therefore$ point of discontinuity (hole) @ $(-5, -10)$

- Factor (if possible)
- Divide out common factors, set common factor equal to 0 and solve for x-value of hole
- Solve remaining function for corresponding y-value

Ex. 1) Determine any non-permissible values

a) $y = \frac{x^2}{x^2-9}$

$y = \frac{x^2}{(x+3)(x-3)}$

V. A. $(x+3)(x-3) = 0$

$x = -3 \quad x = 3$

b) $y = \frac{x^2-x-2}{x+1}$

$y = \frac{\cancel{(x+1)}(x-2)}{\cancel{x+1}}$

$x+1 = 0$

$x = -1$

hole @ $(-1, -3)$

$y = x-2$

$y = -1-2$

$y = -3$

Horizontal Asymptotes (H.A.)

For $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ have no common factors, the following happens:

numerator denominator

- If the degree of $p(x) <$ degree of $q(x)$ the H.A. is the line $\boxed{y = 0}$

$1 < 2$

$y = \frac{x+2}{x^2+6x-7}$

$\therefore y = 0$

num denom

- If the degree of $p(x) =$ to the degree of $q(x)$ then the H.A. is the line $y = \frac{a}{b}$, where "a" is the leading coefficient of $p(x)$ and "b" is the leading coefficient of $q(x)$

$y = \frac{2x^2-1}{3x^2+5}$

H. A.

$y = \frac{a}{b}$

$y = \frac{2}{3}$

num denom

- If the degree of $p(x) >$ the degree of $q(x)$ then there will be no H.A.

$3 > 2$

$y = \frac{x^3}{x^2+3}$

$\therefore$ no H.A.

Ex. 2) Determine the equations of any asymptotes and note the coordinates of any holes.

a) $y = \frac{2x}{x^2-4}$

$y = \frac{2x}{(x-2)(x+2)}$

V.A.

$(x-2)(x+2) = 0$

$x-2=0 \quad x+2=0$

$x=2 \quad x=-2$

H.A.

$1 < 2$

$\therefore y=0$

Note: no common factors so no hole exists

b) $y = \frac{x^2+2x}{x^2-4}$

$y = \frac{x\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$

hole

$x+2=0$

$x=-2$

* must find hole first

$y = \frac{x}{x-2}$

$y = \frac{-2}{-2-2}$

$y = \frac{1}{2}$

$\therefore$ hole @ $(-2, \frac{1}{2})$

$y = \frac{x}{x-2}$ ← find asymptotes

V.A.

$x-2=0$

$x=2$

H.A.

$y=1$

Ex. 3) Solve.

$0 = \frac{3}{x-2} + 4$

restrictions

$x \neq 2$

$-4 = \frac{3}{x-2}$

$-4(x-2) = 3$

$-4x + 8 = 3$

$-4x = -5$

$x = \frac{5}{4}$ ✓

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no graphs